

CZY ZALEŻNOŚCI W UKŁADZIE WIELOKANAŁOWYM MOŻNA BADAĆ PARAMI?

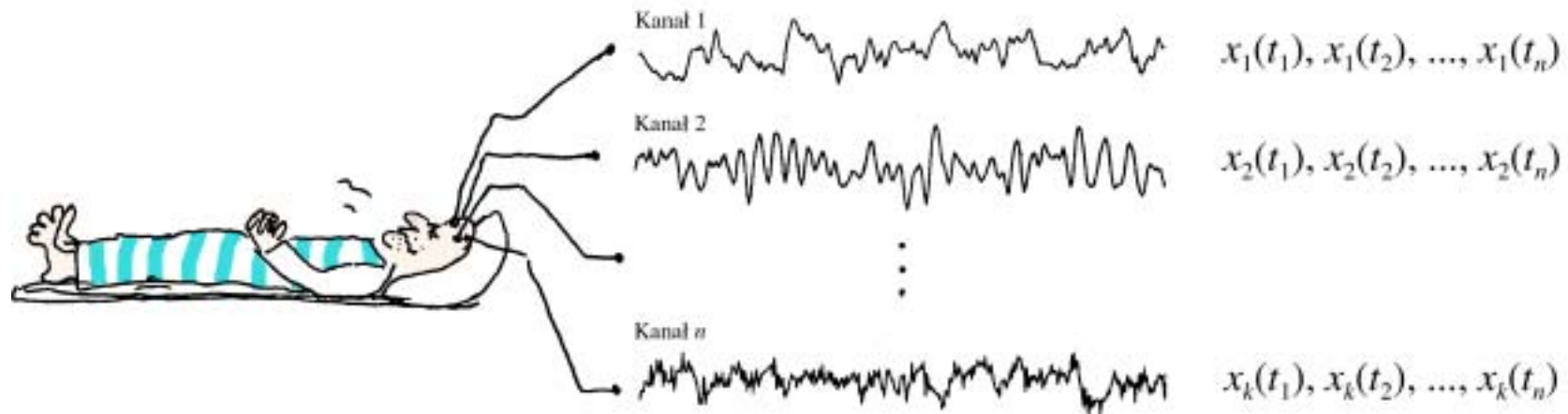
Maciej Kamiński

Pracownia Fizyki Medycznej
Instytut Fizyki Doświadczalnej



Uniwersytet Warszawski

Dane neurobiologiczne



Analiza danych



W zmierzonym kanale:

- amplituda
- obecność pewnych struktur
- skład częstotliwościowy (widmo)
- parametr Ω
- ...

Więcej niż jeden kanał

na przykład dwa kanały:

W kanale 1:

- amplituda
- obecność pewnych struktur
- skład częstotliwościowy (widmo)
- parametr Ω
- ...

W kanale 2:

- amplituda
- obecność pewnych struktur
- skład częstotliwościowy (widmo)
- parametr Ω
- ...

Ale to nie wszystko

Układ dwukanałowy to nie tylko kanał 1 i kanał 2!



To również zależności między nimi:

- widmo wzajemne
- korelacje wzajemne
- koherencje
- ...
- kierunek wpływu jednego na drugi



Channel 1	Channel 2
— amplitude	— amplitude
— spectrum	— spectrum
— ...	— ...

and

(cross)correlation

$$R = \frac{1}{N} \sum_{i=1}^N X_i Y_i$$

(cross)spectrum — spectrum is now a matrix with off-diagonal elements S_{xy} and S_{yx}

$$\begin{pmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{pmatrix}$$

describes common power

(ordinary) coherence

$$K_{ij}^2 = \frac{|S_{ij}|^2}{S_{ii} S_{jj}}$$

describes how much of common signal is in given pair of channels at given frequency

Historia określania kierunku wpływu jednego sygnału na drugi (krótki kurs)

- pierwsze próby — lata 50 (nauki społeczne i ekonomiczne)
 - faza koherencji
 - koherencje skierowane
 - przyczynowość Grangera
 - pętle sprzężeń zwrotnych
 - inne
-
- miary wielokanałowe: DTF, PDC

There is no information about the direction of the influence

What can we do?

Let us first take a closer look at the correlation.

The correlation can be calculated for time shifted signals (for certain *lag*)

$$R(s) = \frac{1}{N-|s|} \sum_{i=1}^{N-|s|} X_i Y_{i-s}$$

and

may show a peak at certain nonzero lag!

(cross-spectrum/coherence may have certain nonzero phase)

Problems — in real brutal life the common situation is:

- maximum of cross-correlation is at (or very close to) lag 0
- there is no clear maximum at all

In the spectral domain the typical problem is the ambiguity of the phase (modulo 2π) which makes clear direction description impossible

People started thinking about that problems long time ago.
The first insights came from social sciences about 1950's.
These theories are a basis for methods called today "structural equations modelling". The idea is as follows: we have several signals, iterdependent between each other.

We must first have certain knowledge about the investigated variables and relations between them.

Using a priori knowledge we attribute some directions for connections between them and then we test our assumptions. By for instance regressing one signal against another we see dependency then we assume that the direction is as attributed before.

This is not an objective method and results depend on our previous assumptions. In many cases these methods do not use time (temporal order) in calculations making interpretation of the results very difficult.

For neurobiologists such an approach has no value but some people try to use them anyway.

The importance of temporal ordering in the objective inference of causal relations was first recognized by Wiener

Wiener N. *The theory of prediction* Modern Mathematics for Engineers, Series I, E. F. Beckenback ed., Chapter 8 (1956)

He introduced the notion of causality in a following way:

For two simultaneously measured signals — if we can better predict the first signal using the past information from the second one than using the information without it then we call the second signal causal to the first one.

This very general definition was adopted by Granger

Granger C. W. J. *Investigating causal relations by econometric models and cross-spectral methods* *Econometrica* **37**, 907-915 (1969)

He restricted the general Wiener definition to the form of linear models of time series relations.

He also defined basic terms like causality, instantaneous causality, causality lag, causality coherence.

Econometrica, Vol. 37, No. 3 (July, 1969)

**INVESTIGATING CAUSAL RELATIONS BY ECONOMETRIC MODELS
AND CROSS-SPECTRAL METHODS**

BY C. W. J. GRANGER

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The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2003

"for methods of analyzing economic time series with time-varying volatility (ARCH)"

"for methods of analyzing economic time series with common trends (cointegration)"

Robert F. Engle
 1/2 of the prize
 USA

New York University
 New York, NY, USA
 b. 1942

Clive W.J. Granger
 1/2 of the prize
 United Kingdom

University of California
 San Diego, CA, USA
 b. 1934

The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2003
 Prize Announcement
 Press Release
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 Presentation Speech

Robert F. Engle
 Prize Lecture
 Interview
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 Prize Award Photo
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Clive W.J. Granger
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4. CAUSALITY

Cross-spectral methods provide a useful way of describing the relationship between two (or more) variables when one is causing the other(s). In many realistic economic situations, however, one suspects that feedback is occurring. In these situations the coherence and phase diagrams become difficult or impossible to interpret, particularly the phase diagram. The problem is how to devise definitions of causality and feedback which permit tests for their existence. Such a definition was proposed in earlier papers [4, Chapter 7; 3]. In this section, some of these definitions will be discussed and extended. Although later sections of this paper will use this definition of causality they will not completely depend upon it. Previous papers concerned with causality in economic systems [1, 6, 7, 8] have been particularly concerned with the problem of determining a causal interpretation of simultaneous equation systems, usually with instantaneous causality. Feedback is not explicitly discussed. This earlier work has concentrated on the form that the parameters of the equations should take in order to discern definite causal relationships. The stochastic elements and the natural time ordering of the variables play relatively minor roles in the theory. In the alternative theory to be discussed here, the stochastic nature of the variables and the direction of the flow of time will be central features. The theory is, in fact, not relevant for nonstochastic variables and will rely entirely on the assumption that the future cannot cause the past. This theory will not, of course, be contradictory to previous work but there appears to be little common ground. Its origins may be found in a suggestion by Wiener [9]. The relationship between the definition discussed here and the work of Good [2] has yet to be determined.

If A_t is a stationary stochastic process, let \bar{A}_t represent the set of past values $\{A_{t-j}, j = 1, 2, \dots, \infty\}$ and \bar{A}_t represent the set of past and present values $\{A_{t-j}, j = 0, 1, \dots, \infty\}$. Further let $\bar{A}(k)$ represent the set $\{A_{t-j}, j = k, k + 1, \dots, \infty\}$.

Denote the optimum, unbiased, least-squares predictor of A_t using the set of values B , by $P_t(A|B)$. Thus, for instance, $P_t(X|\bar{X})$ will be the optimum predictor of X_t using only past X_t . The predictive error series will be denoted by $e_t(A|B) = A_t - P_t(A|B)$. Let $\sigma^2(A|B)$ be the variance of $e_t(A|B)$.

The initial definitions of causality, feedback, and so forth, will be very general in nature. Testable forms will be introduced later. Let U_t be all the information in the universe accumulated since time $t - 1$ and let $U_t - Y_t$ denote all this information apart from the specified series Y_t . We then have the following definitions.

DEFINITION 1: Causality. If $\sigma^2(X|U) < \sigma^2(X|\bar{U} - \bar{Y})$, we say that Y is causing X , denoted by $Y_t \Rightarrow X_t$. We say that Y_t is causing X_t if we are better able to predict X_t using all available information than if the information apart from Y_t had been used.

DEFINITION 2: Feedback. If

$$\begin{aligned}\sigma^2(X|\bar{U}) &< \sigma^2(X|\bar{U} - \bar{Y}), \\ \sigma^2(Y|\bar{U}) &< \sigma^2(Y|\bar{U} - \bar{X}),\end{aligned}$$

we say that feedback is occurring, which is denoted $Y_t \Leftrightarrow X_t$, i.e., feedback is said to occur when X_t is causing Y_t and also Y_t is causing X_t .

PRZYCZYNOWOŚĆ GRANGERA¹ bazuje na przewidywalności sygnałów:

Jeśli jeden sygnał $X_1(t)$ zawiera w swoich poprzednich próbkach informację, która pomaga przewidzieć drugi sygnał $X_2(t)$, wtedy mówimy, że $X_1(t)$ jest sygnałem źródłowym dla $X_2(t)$.

Jak widać jest to wielkość oryginalnie zdefiniowana dla dwóch kanałów. Można rozszerzyć to pojęcie również dla układów wielokanałowych².



¹ C. W. J. Granger, *Investigating causal relations by econometric models and cross-spectral methods*. *Econometrica*, 1969, 37:424-438.

² M. Kamiński et al., *Evaluating causal relations in neural systems: Granger causality, directed transfer function and statistical assessment of significance*. *Biol. Cybern.* 2001, 85:145-157.

Let us assume the two signals measured simultaneously in discrete time intervals

$$X(t) = \sum_{j=1}^p A'_{11}(j)X(t-j) + \varepsilon'(t)$$

$$X(t) + A_{12}(0)Y(t) = \sum_{j=1}^p A_{11}(j)X(t-j) + \sum_{j=1}^p A_{12}(j)Y(t-j) + \varepsilon(t)$$

$$Y(t) + A_{21}(0)X(t) = \sum_{j=1}^p A_{21}(j)X(t-j) + \sum_{j=1}^p A_{22}(j)Y(t-j) + \eta(t)$$

If the variance of prediction error $\varepsilon(t)$ is reduced by including the knowledge of past (and maybe present) values of Y_t to the relation for X_t , then we say that Y_t Granger causes X_t . In other words we may say that A_{12} are greater than 0.

Różne podejścia do obliczania widma sygnału

- Metody nieparametryczne
np. transformata Fouriera
- Metody parametryczne
np. model autoregresyjny (AR)

Model AR

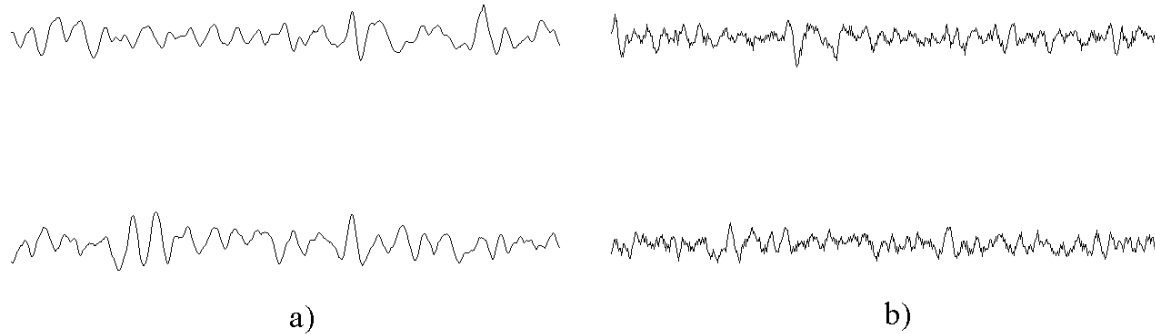
Zakładamy, że posiadany przez nas sygnał możemy opisać wzorem

$$x(t) = \sum_{i=1}^p a_i x(t-i) + e(t)$$

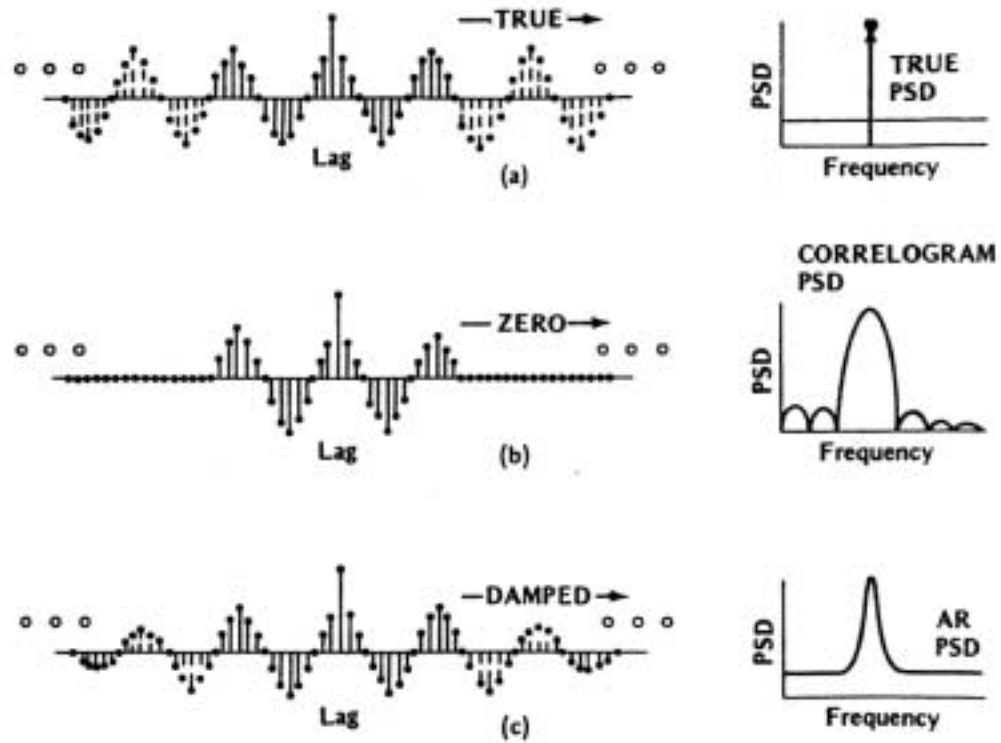
Wartość sygnału x w chwili t zależy od p jego wartości poprzednich z dodanym pewnym składnikiem losowym.

Jeżeli nasze dane dobrze pasują do takiego modelu, całą wiedzę o sygnale możemy czerpać tylko ze współczynników modelu (parametrów a_i).

Okazuje się, że wiele sygnałów biologicznych dobrze opisuje się w taki sposób.



Jakie są zalety użycia modelu AR?



Z: S. Marple *Digital spectral analysis with applications*

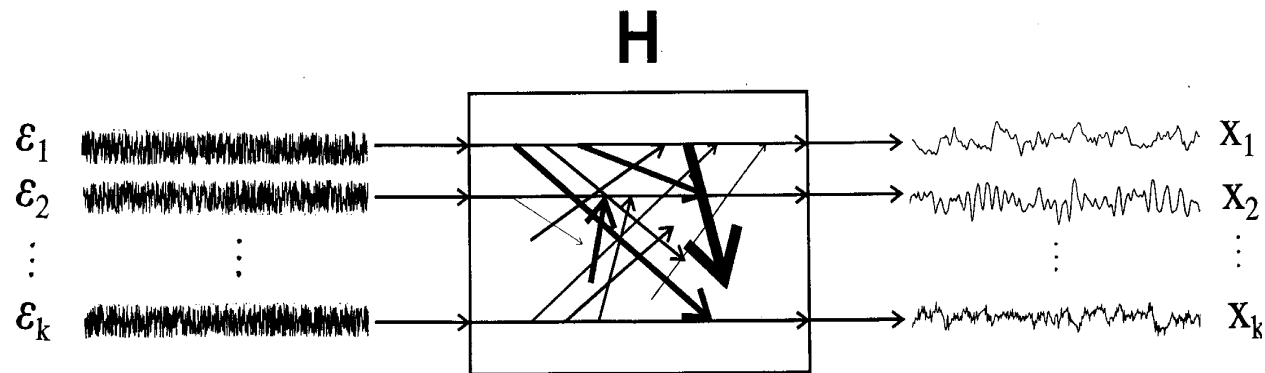
Model AR jako filtr liniowy

$$\mathbf{X}(t) = \sum_{i=1}^p \mathbf{A}(i)\mathbf{X}(t-i) + \mathbf{E}(t)$$

$$\sum_{i=0}^p \mathbf{A}(i)\mathbf{X}(t-i) = \mathbf{E}(t)$$

$$\mathbf{A}(f)\mathbf{X}(f) = \mathbf{E}(f)$$

$$\mathbf{X}(f) = \mathbf{A}^{-1}(f)\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f)$$



Ze współczynników modelu AR możemy obliczyć widmo

$$\mathbf{S}(f) = \mathbf{X}(f)\mathbf{X}^*(f) = \dots = \mathbf{H}(f)\mathbf{V}\mathbf{H}^*(f)$$

koherencje (zwyczajne)

$$K_{ij}^2(f) = \frac{S_{ij}^2(f)}{S_{ii}(f)S_{jj}(f)}$$

Oraz wiele innych funkcji, na przykład...

Kierunkową Funkcję Przejścia (Directed Transfer Function)

DTF

opisującą transmisję sygnału z kanału
 i do kanału j w dziedzinie częstości

wersja nienormalizowana $\theta_{ij}^2(f) = |H_{ij}(f)|^2$

wersja normalizowana do
wpływów do kanału i $\gamma_{ij}^2(f) = \frac{|H_{ij}(f)|^2}{\sum_{m=1}^k |H_{im}(f)|^2}$

Biol. Cybern. 65, 203–210 (1991)

**Biological
Cybernetics**
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A new method of the description of the information flow in the brain structures

M. J. Kamiński and K. J. Blinowska

Medical Physics Laboratory, Institute of Experimental Physics, Warsaw University, ul. Hoża 69, PL-00-681 Warszawa, Poland

Received February 10, 1991

DTF ma interpretację przyczynowości Grangera również w przypadku wielokanałowym.

W związku z tym DTF opisuje transmisję sygnału łącznie przez wszystkie możliwe drogi przepływu, a nie tylko bezpośrednie połączenia między kanałami.

Symulacje wykonał

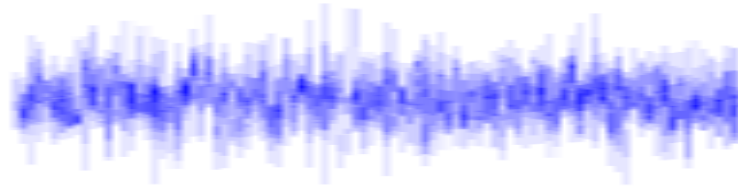


mgr Rafał Kuś

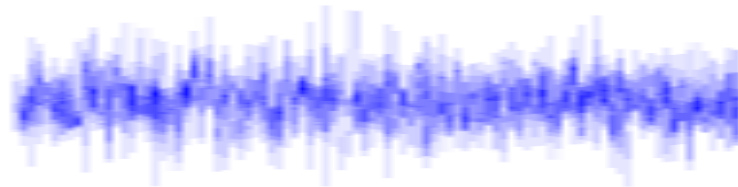
EEG



szum



sygnał w
kan. 1



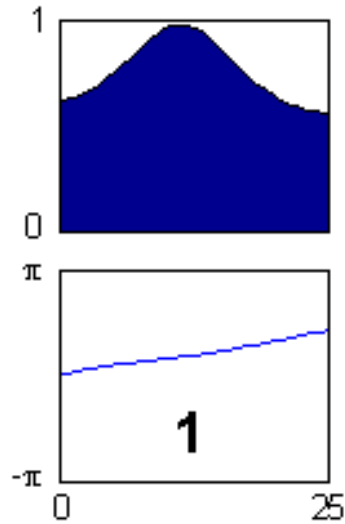
EEG – EEG człowieka z elektrody P3, stan relaksu

Δ – opóźnienie

szum – proces czysto losowy (szum biały)

amplituda szumu/amplituda EEG = 3

EEG



amplituda koherencji

faza koherencji



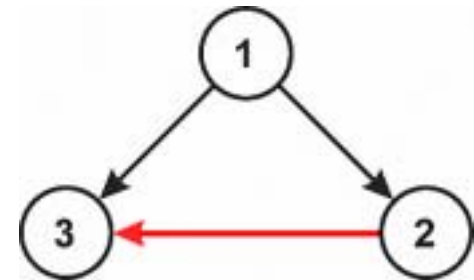
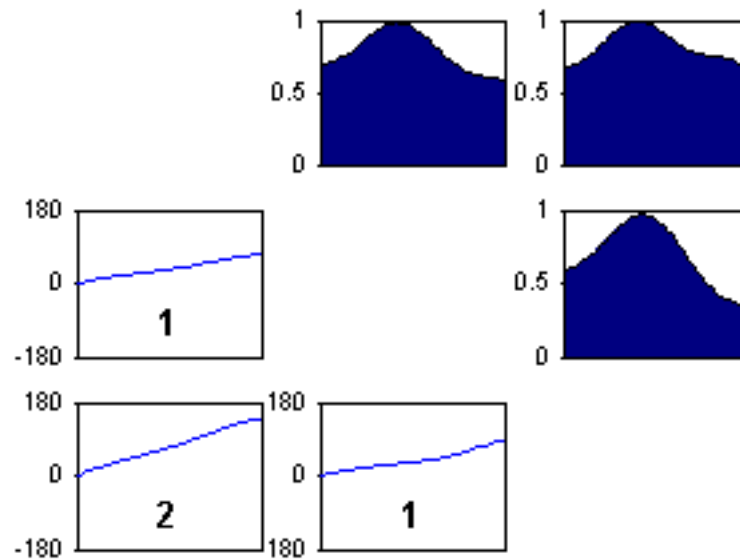
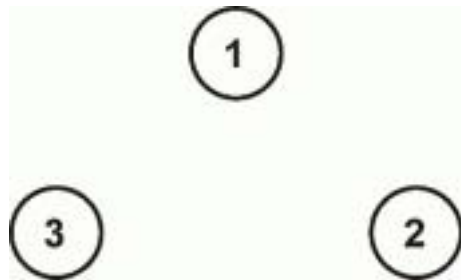
Pierwsze kłopoty

A ja mam trzy kanały...

Czy ta sytuacja różni się czymkolwiek od układu dwukanałowego?

Zaskakująco wielu badaczy nie zdaje sobie sprawy z tego, że różnica taka nie tylko jest, ale jest również bardzo istotna.

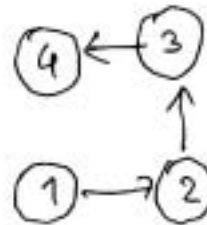
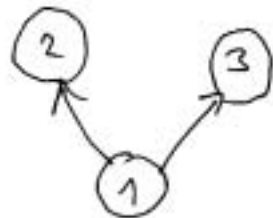
Analiza koherencji



Nowe możliwości

Kiedy system składa się z więcej niż dwóch kanałów pojawiają się w nim nowe możliwości zależności między kanałami.

Do ich opisu potrzebne są nowe funkcje.

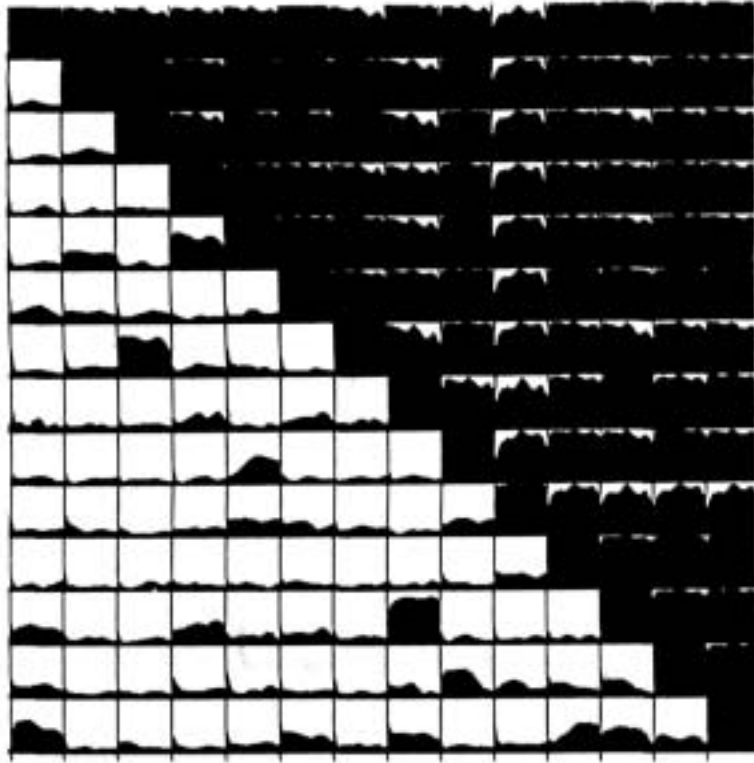


Koherencje cząstkowe — opisują tylko *bezpośrednie* związki między kanałami (kiedy wpływ pozostałych kanałów jest statystycznie pominięty).

$$G_{ij}^2(f) = \frac{\mathbf{M}_{ij}^2(f)}{\mathbf{M}_{ii}(f)\mathbf{M}_{jj}(f)}$$

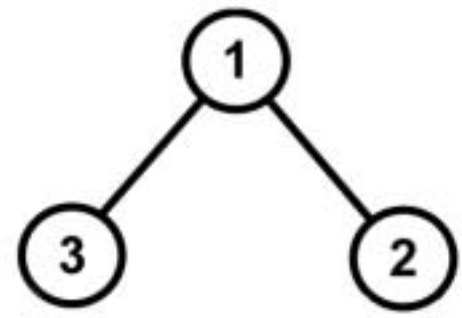
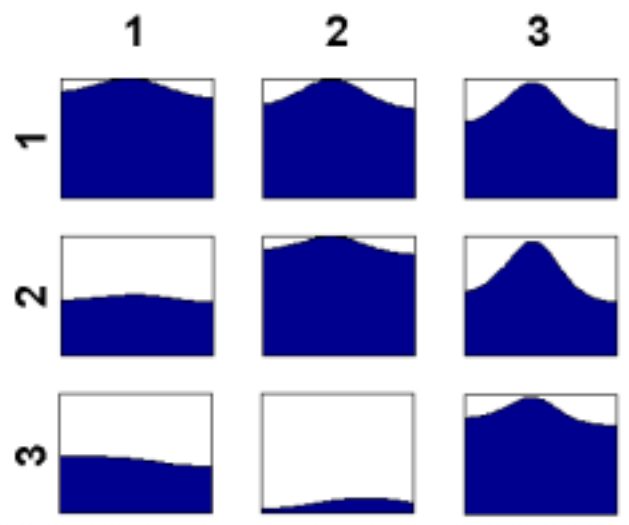
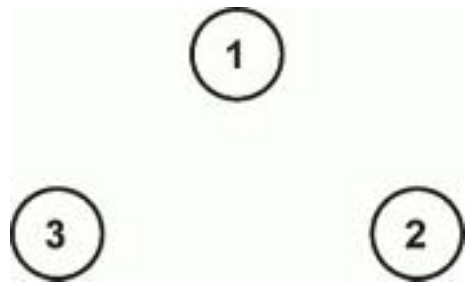
Koherencje wielorakie — opisują związek danego kanału ze wszystkimi innymi kanałami w zestawie.

62 11 49 47 45 2 32 7 21 43 26 23 4 28



28 4 23 26 43 21 7 32 2 45 47 49 11 62

koherencje zwyczajne, cząstkowe i wielorakie



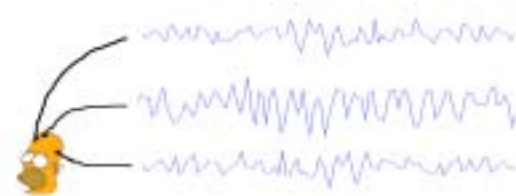
Analiza danych



- miary jednokanałowe (np. widmo)

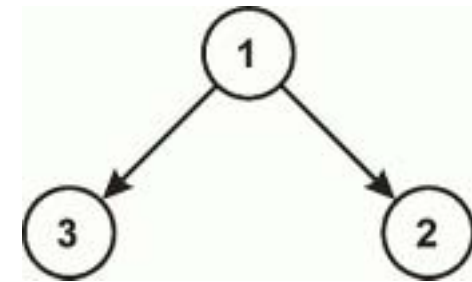
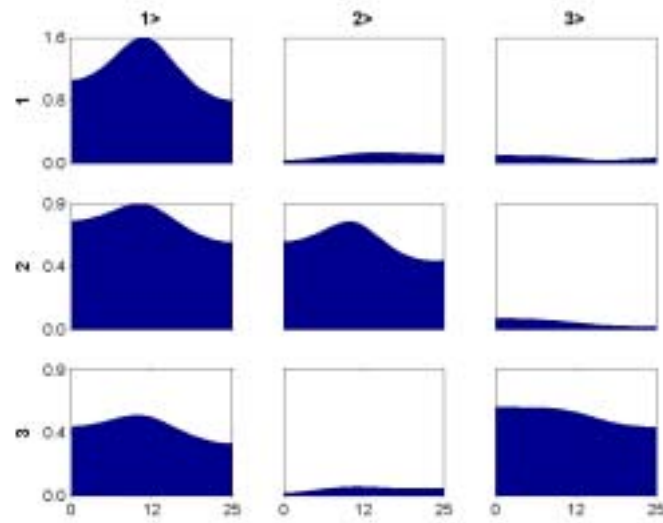
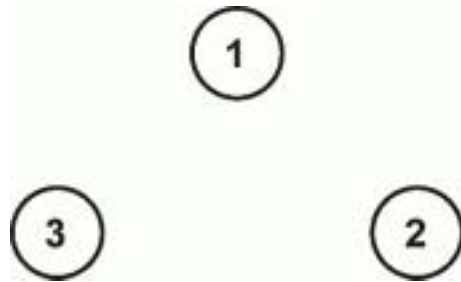


- miary jednokanałowe (np. widmo)
- zależności międzykanałowe (np. koherencje zwyczajne)

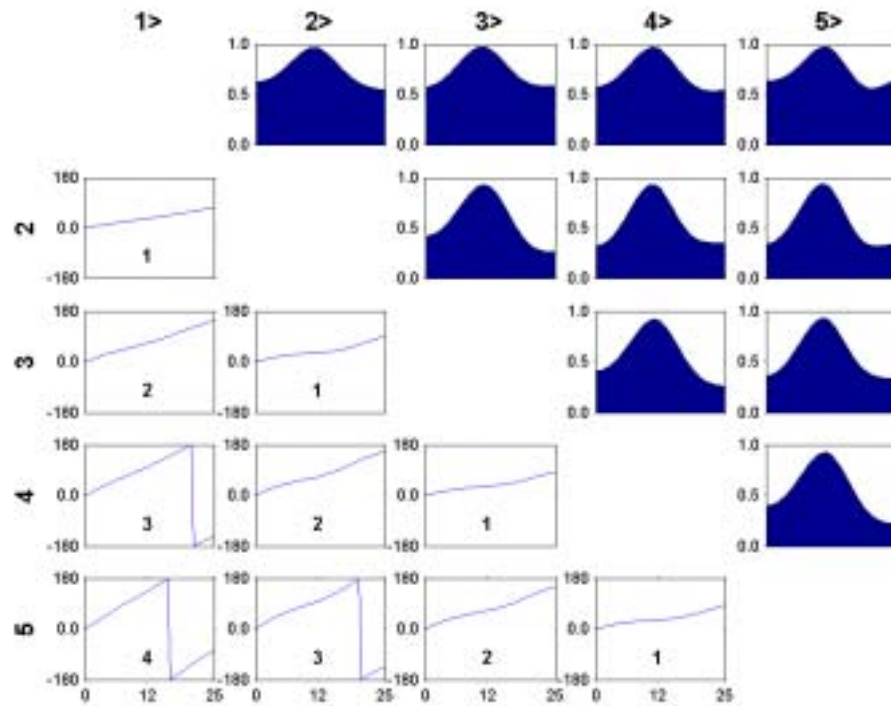


- miary jednokanałowe (np. widmo)
- zależności międzykanałowe (np. koherencje zwyczajne)
- miary cząstkowe (np. koherencje cząstkowe)
- ...

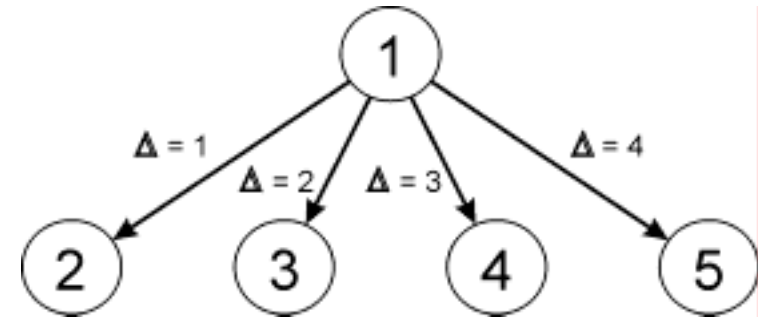
DTF



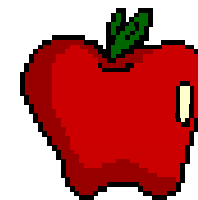
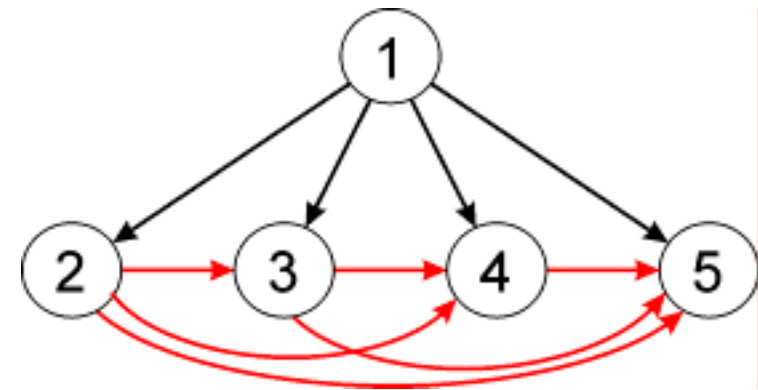
Koherencje (parami)



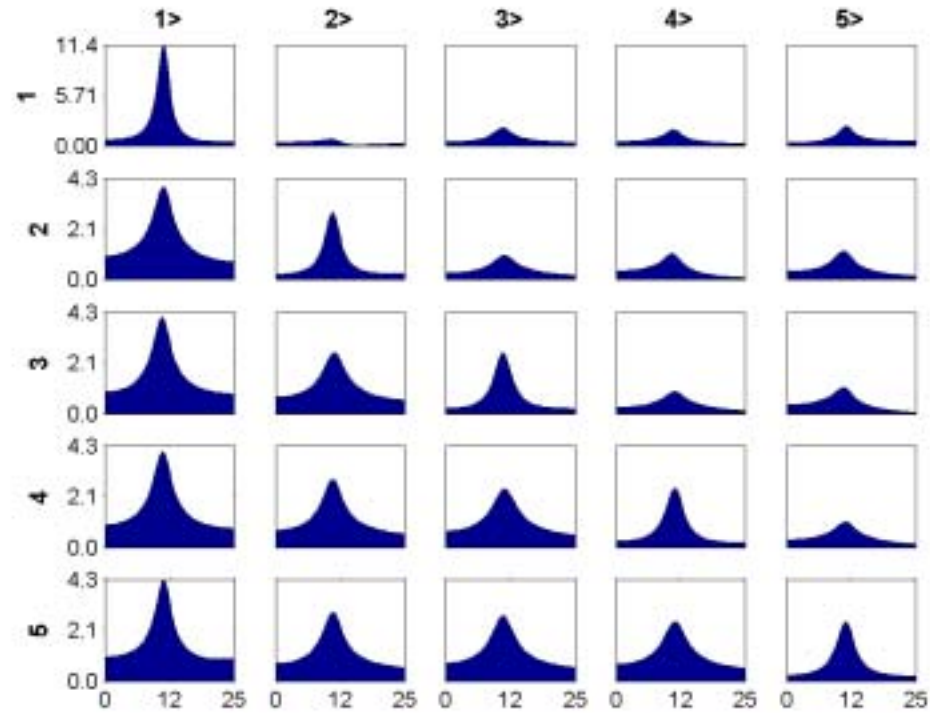
Symulacja



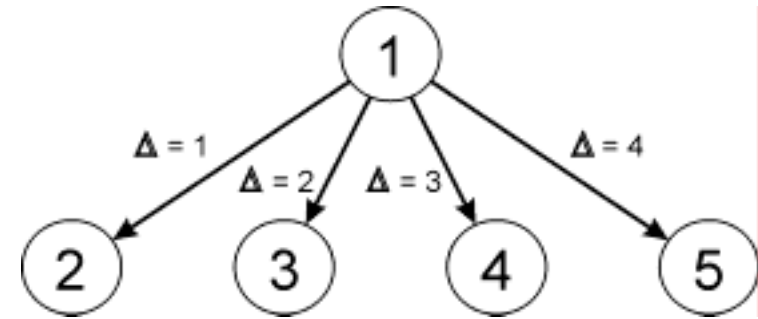
Wynik



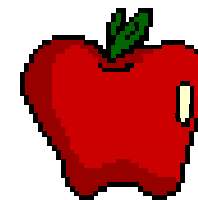
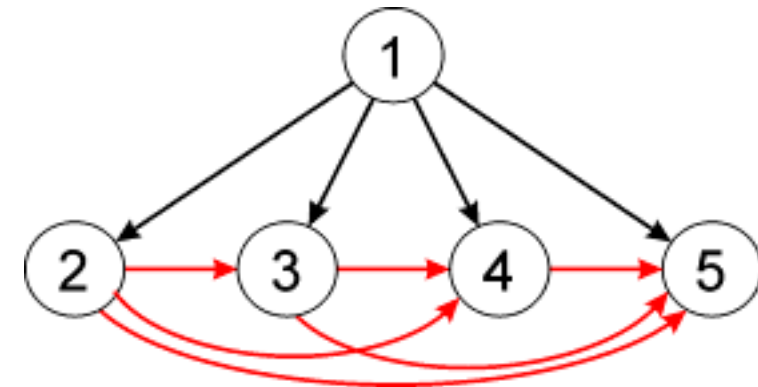
DTF (parami)



Symulacja

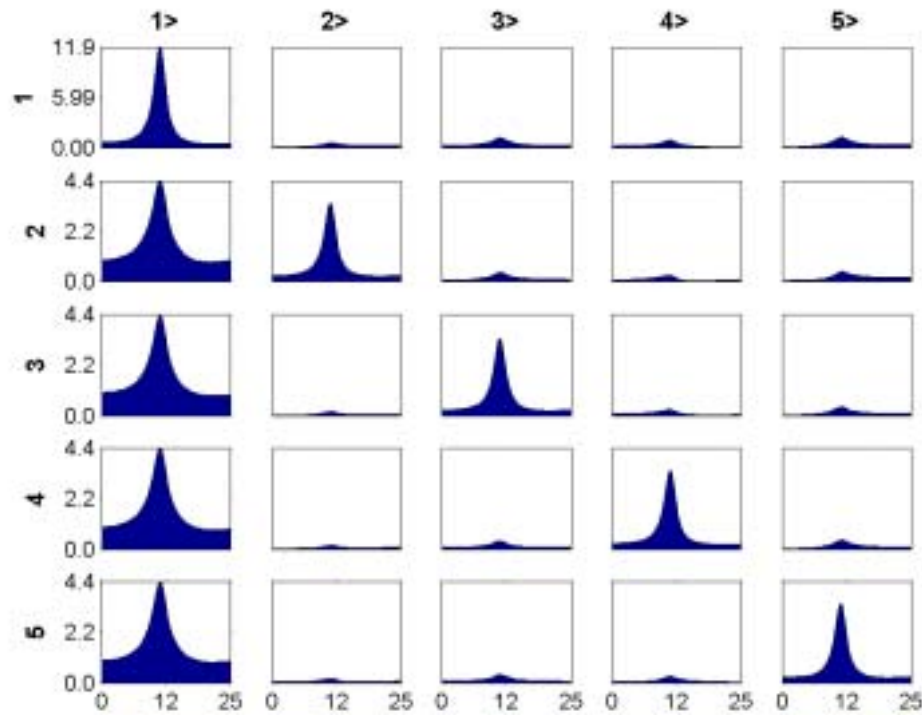
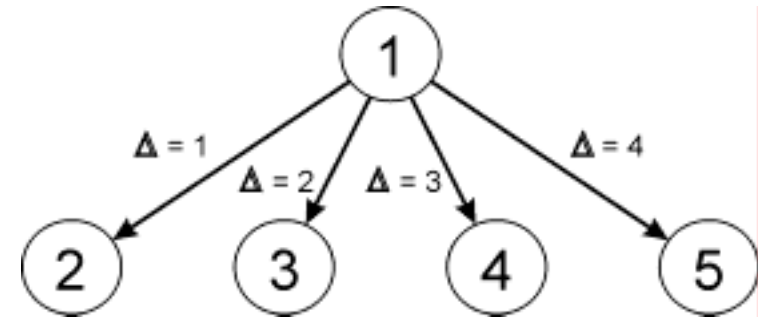


Wynik

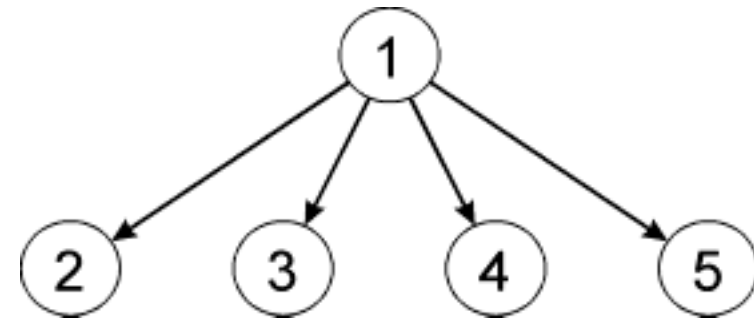


DTF (wielokanałowo)

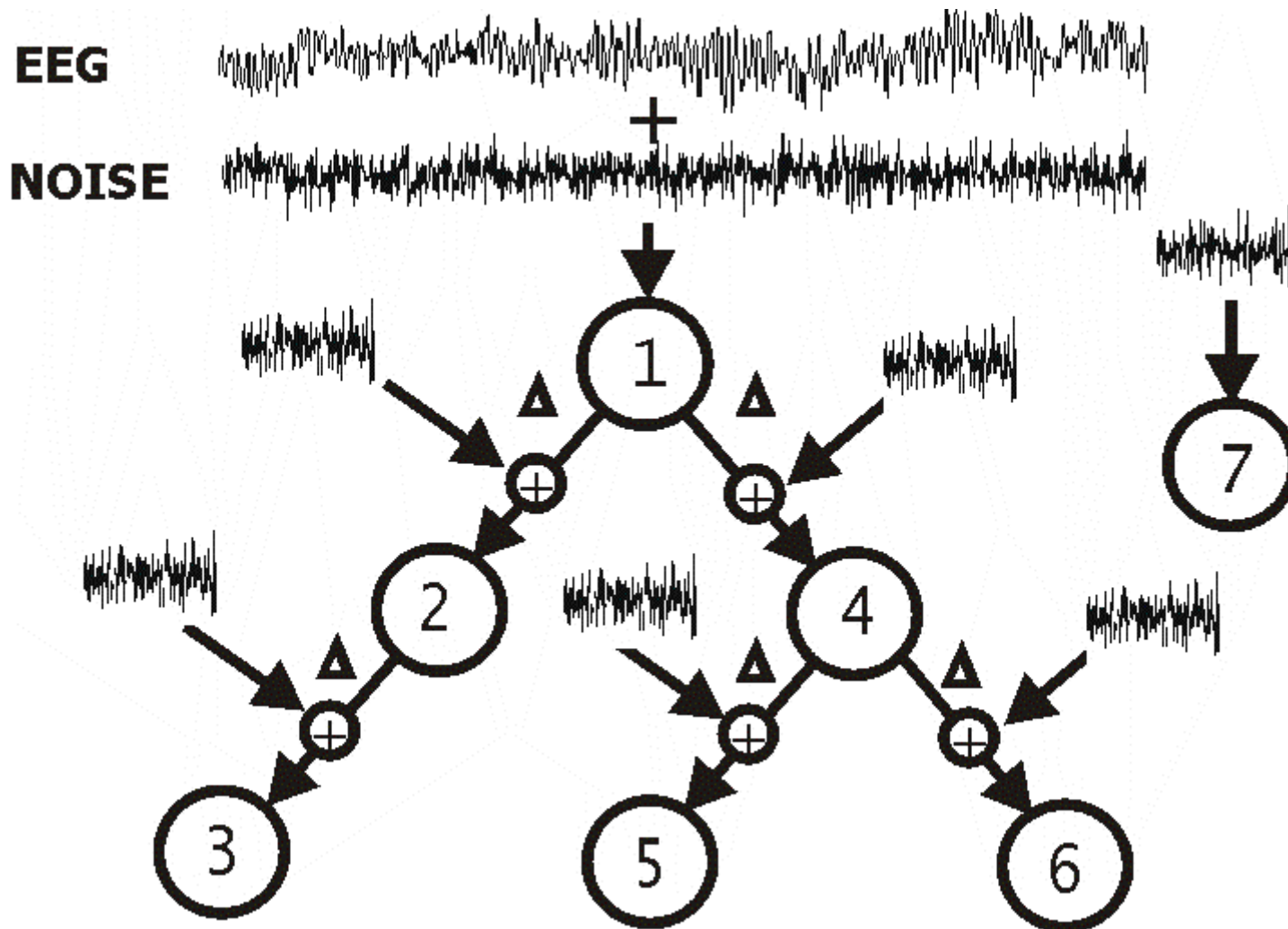
Symulacja



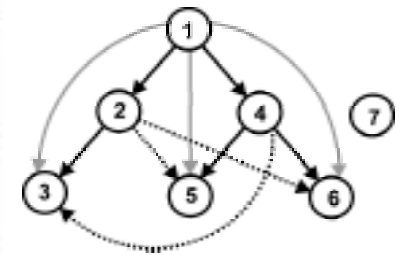
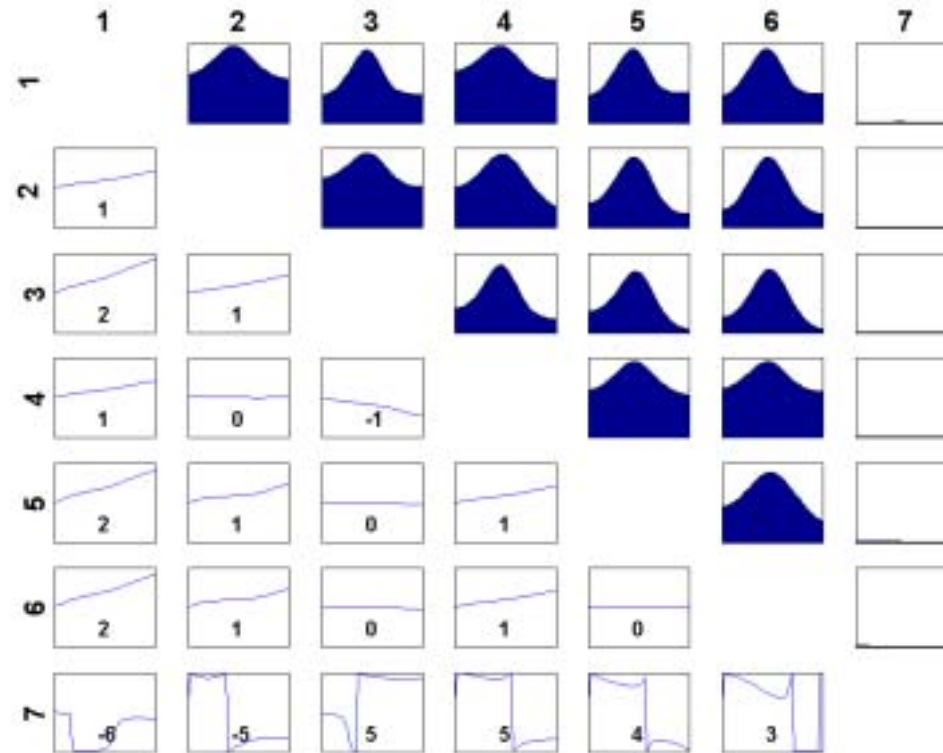
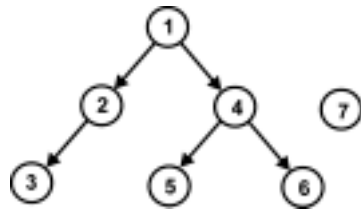
Wynik



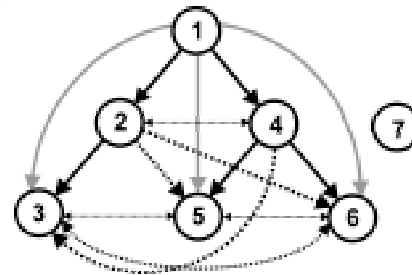
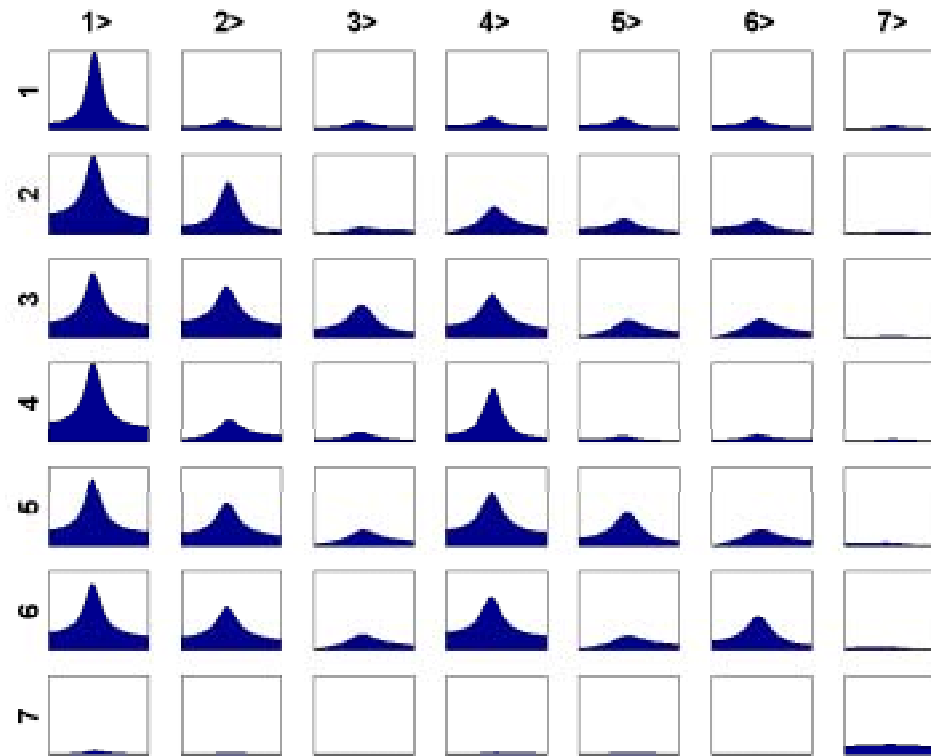
Transmisja przez wiele kanałów

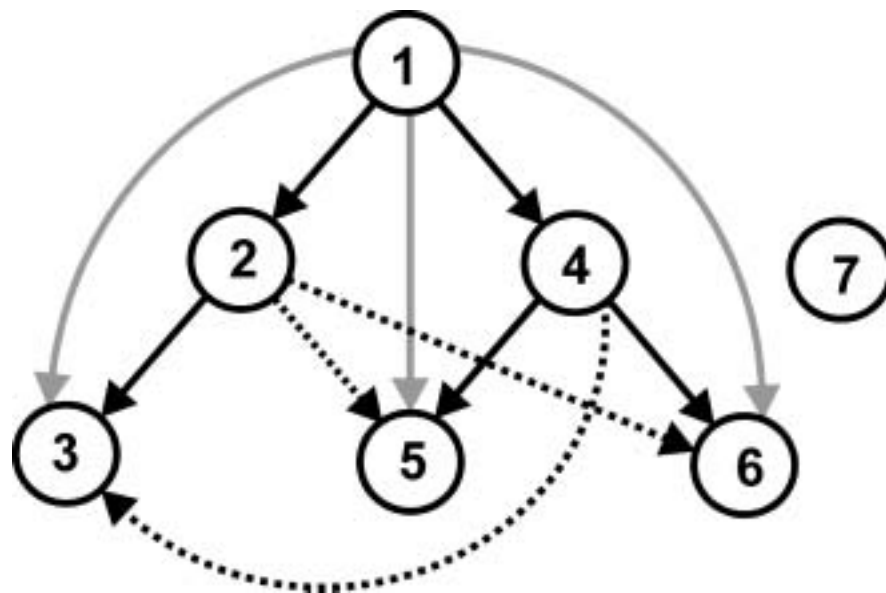
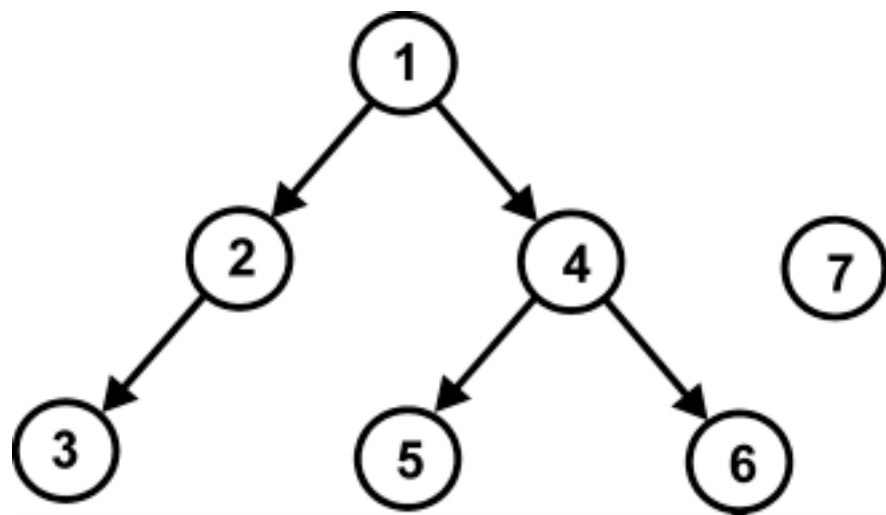


koherencje zwyczajne

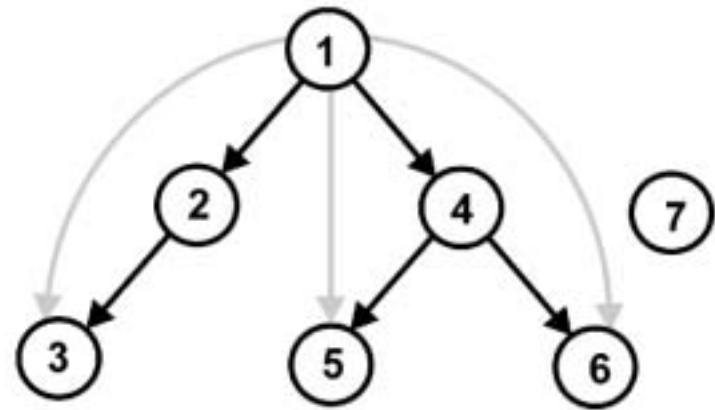
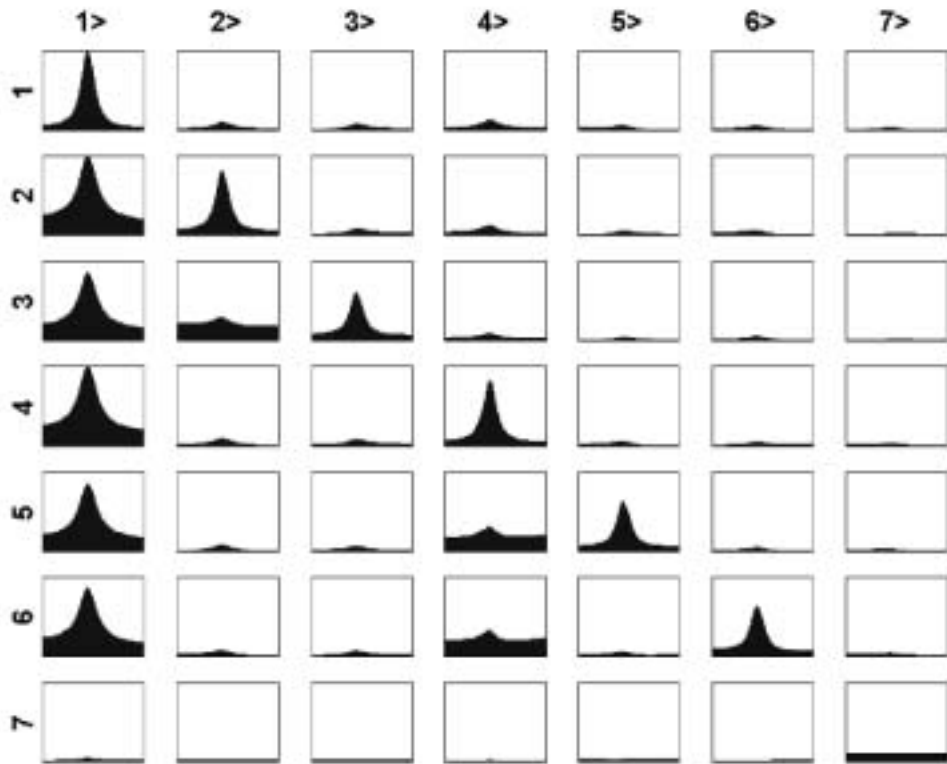


dwukanałowa przyczynowość Grangera





DTF



dDTF — funkcja do opisu przepływów bezpośrednich

$$\text{dDTF} = \text{DTF} \times \text{koherencja cząstkowa}$$



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Determination of information flow direction among brain structures
by a modified directed transfer function (dDTF) method

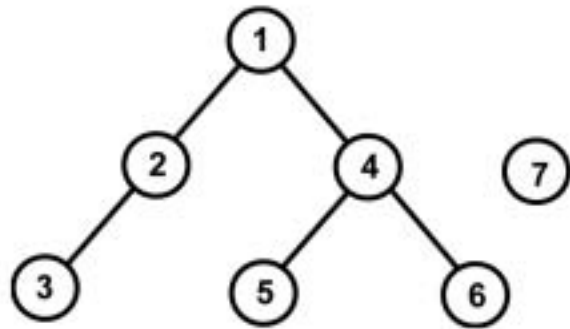
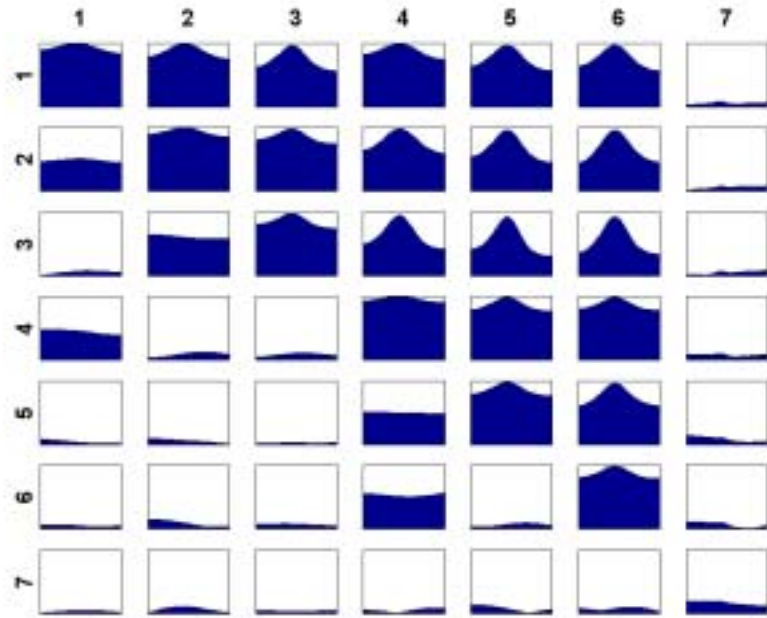
Anna Korzeniewska^a, Małgorzata Mańczak^b, Maciej Kamiński^b,
Katarzyna J. Blinowska^b, Stefan Kasicki^{a,*}

^a *Neurophysiology Department, Nencki Institute of Experimental Biology, Polish Academy of Sciences, 3 Pasteur Street, 02-093 Warsaw, Poland*

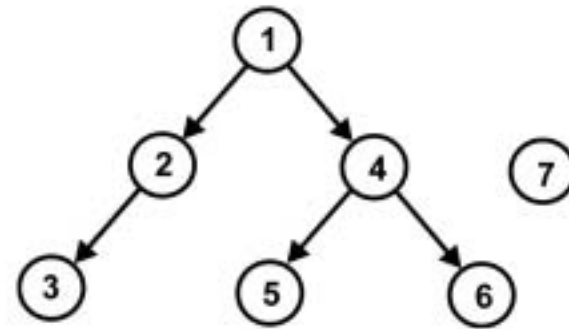
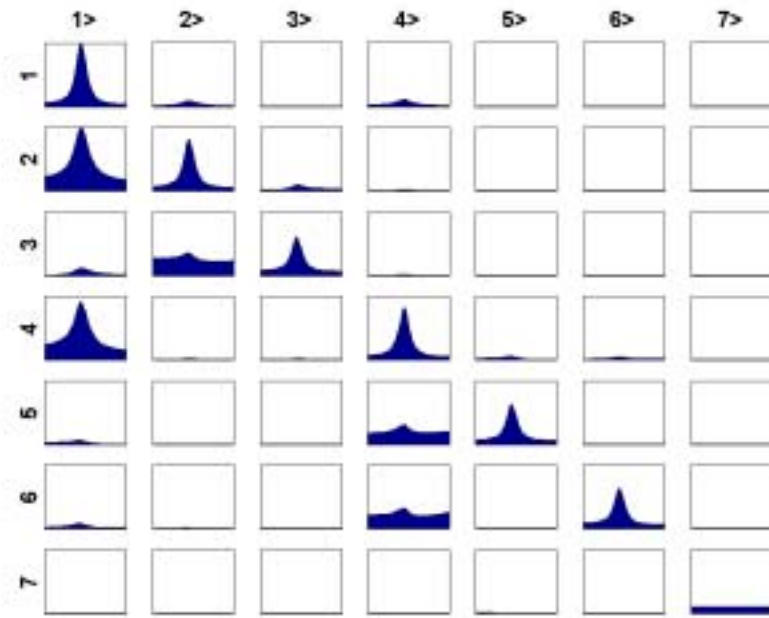
^b *Laboratory of Medical Physics, Warsaw University, 69 Hoża Street, 00-691 Warsaw, Poland*

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koherencje cząstkowe



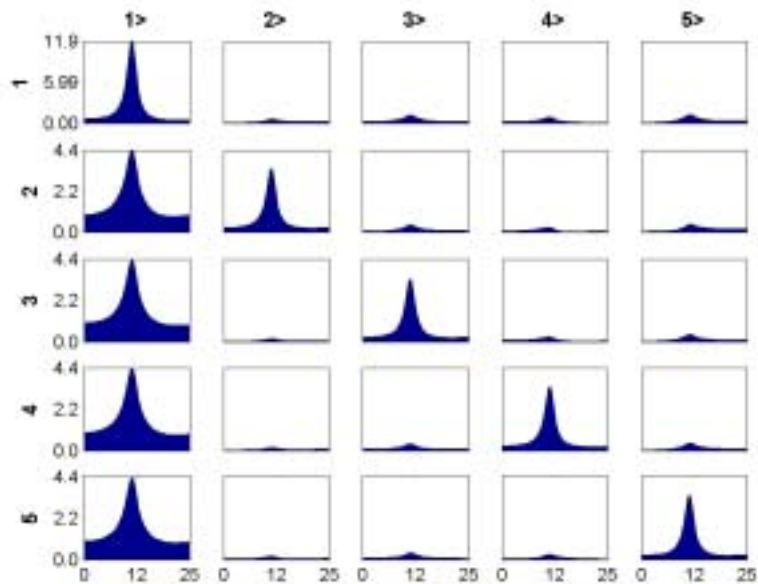
dDTF



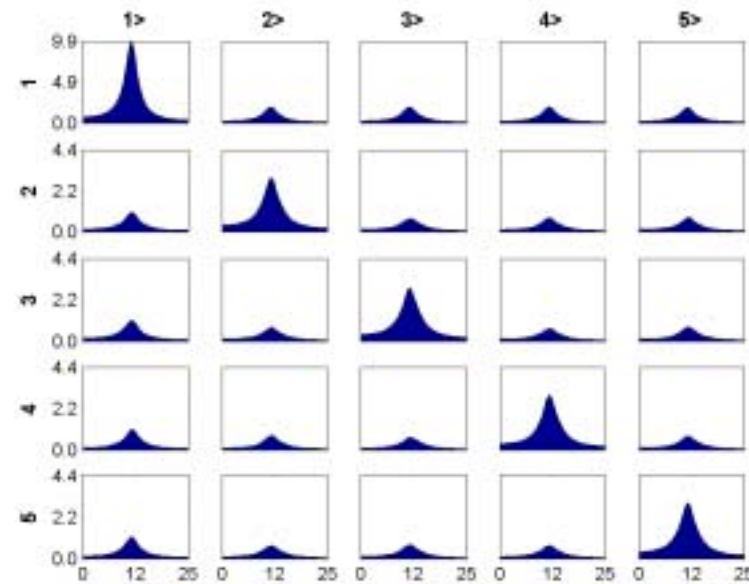
Surrogate data test

The "leak currents" indicating the accuracy of DTF are found from surrogate data – signals with disturbed phases. Surrogate data are obtained by transforming the data to the frequency domain, randomizing their phases and transforming back to the time domain.

MVAR Granger Causality



Surrogate of MVAR Granger Causality





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Electroencephalography and clinical Neurophysiology 102 (1997) 216–227



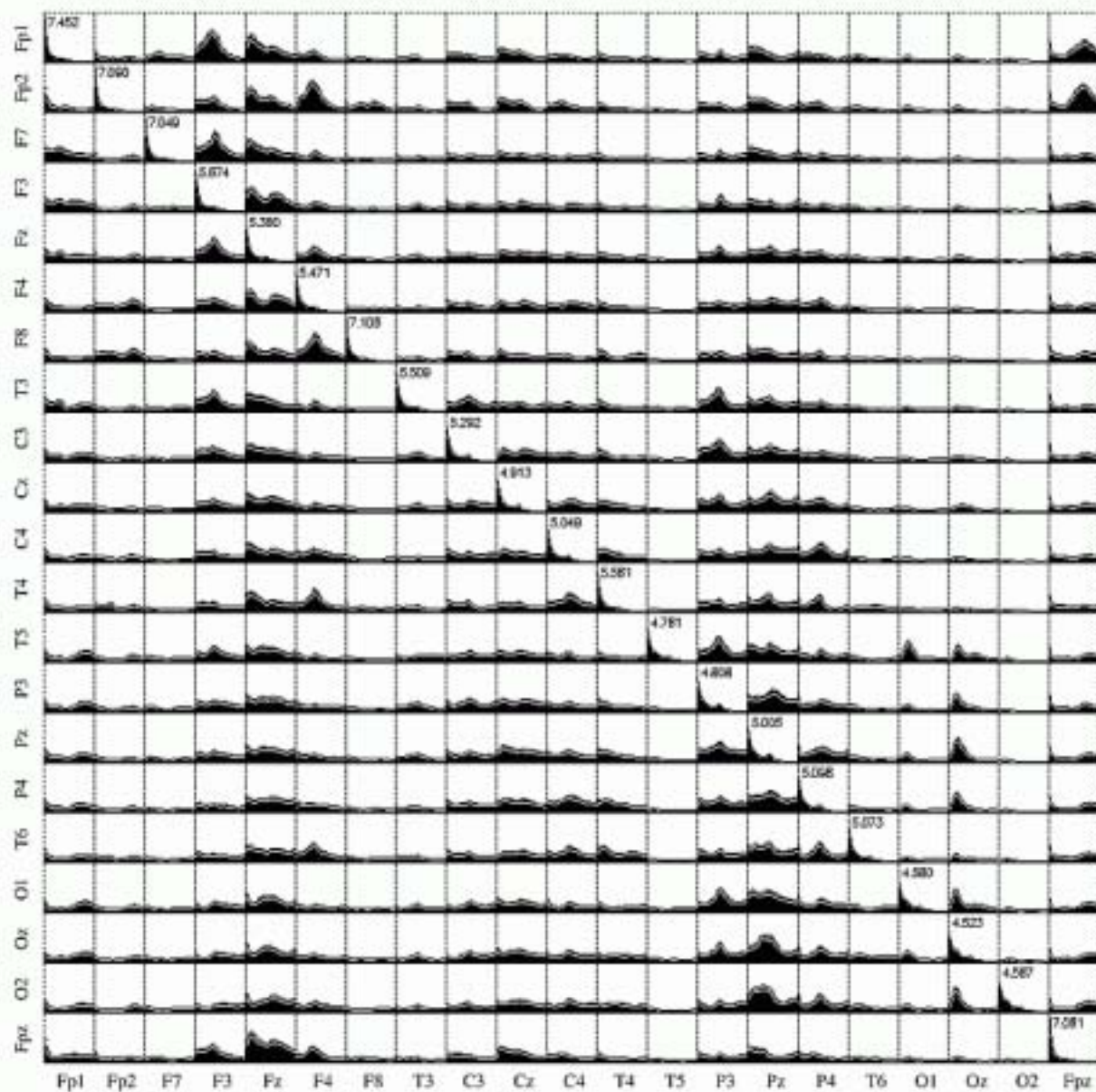
Topographic analysis of coherence and propagation of EEG activity during sleep and wakefulness

Maciej Kamiński^a, Katarzyna Blinowska^{a,*}, Waldemar Szelenberger^b

^a*Laboratory of Medical Physics, Warsaw University, ul. Hoża 69, 00-681 Warsaw, Poland*

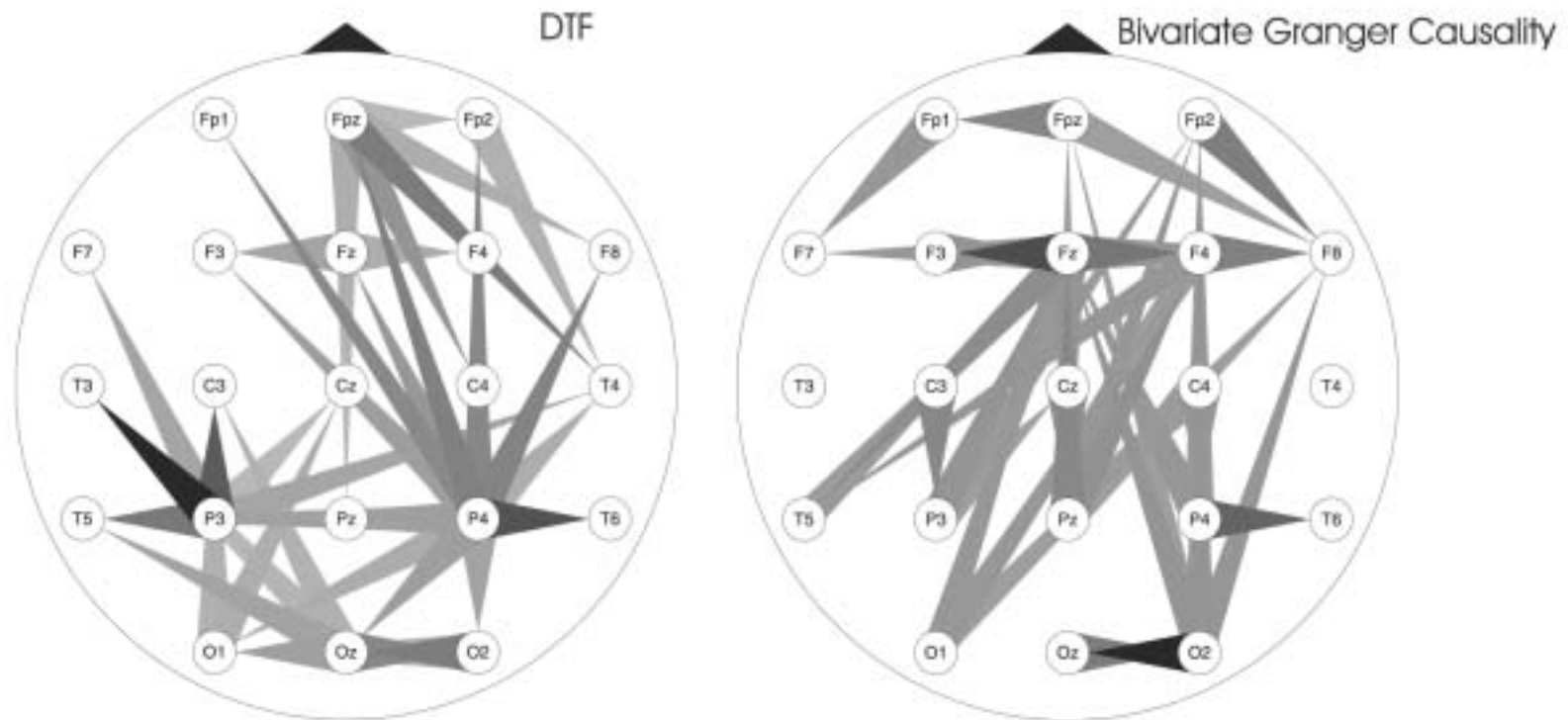
^b*Department of Psychiatry, Warsaw Medical Academy, Warsaw, Poland*

Received 20 December 1995; revised version received 29 May 1996; accepted for publication: 21 August 1996

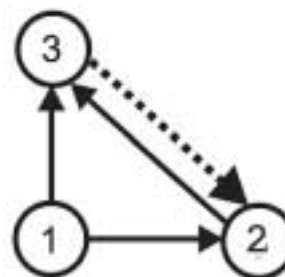
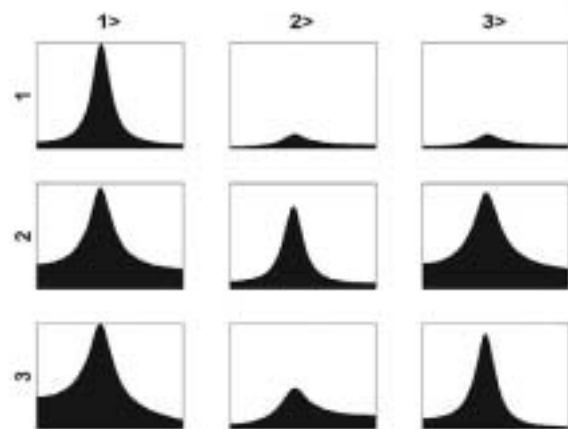
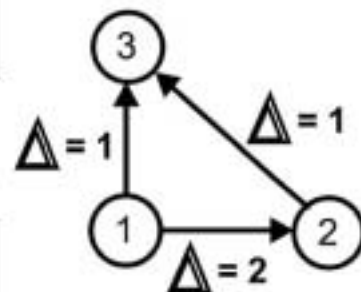
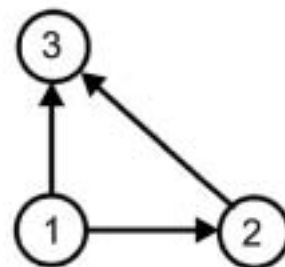
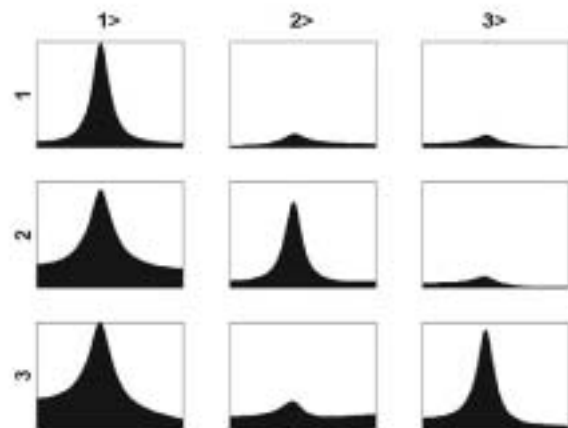


	1	2	3
1	S1	2→1	3→1
2	1→2	S2	3→2
3	1→3	2→3	S3
4	1→4	2→4	3→4
5	1→5	2→5	3→5
6	1→6	2→6	3→6
7	1→7	2→7	3→7
8	1→8

model AR został dopasowany jednocześnie do 21 kanałów EEG ze snu całonocnego (system 10/20) i policzone zostały funkcje DTF.

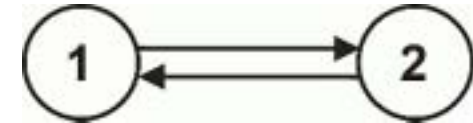
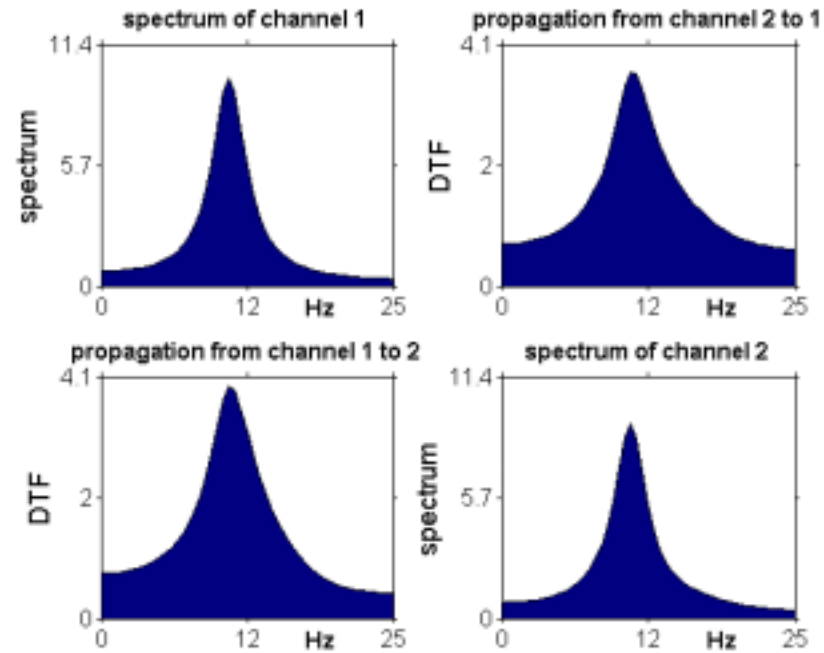


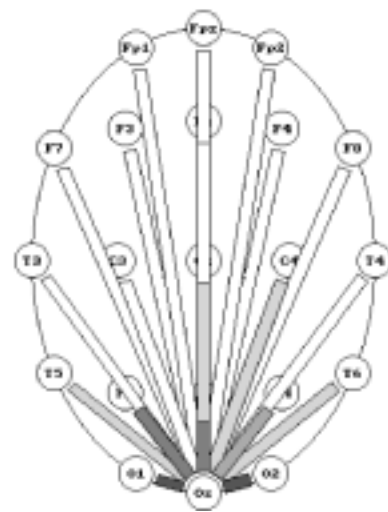
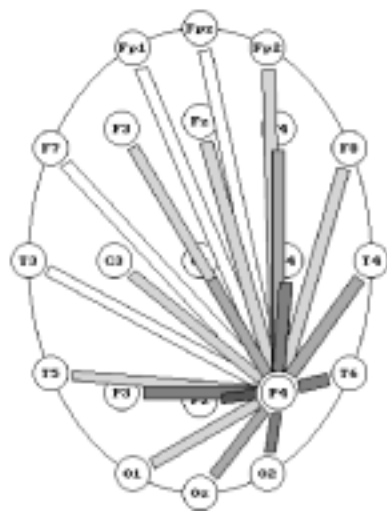
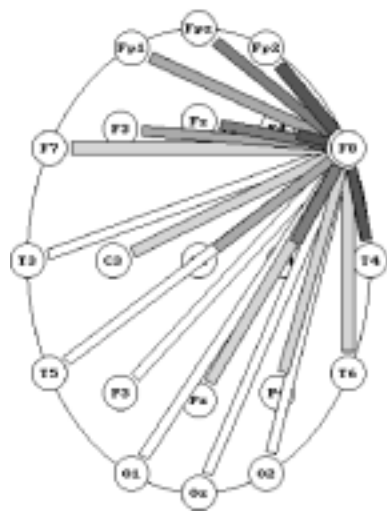
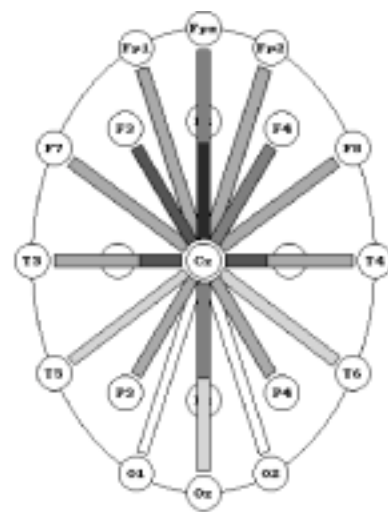
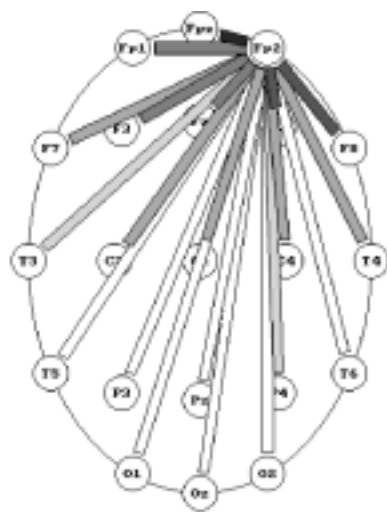
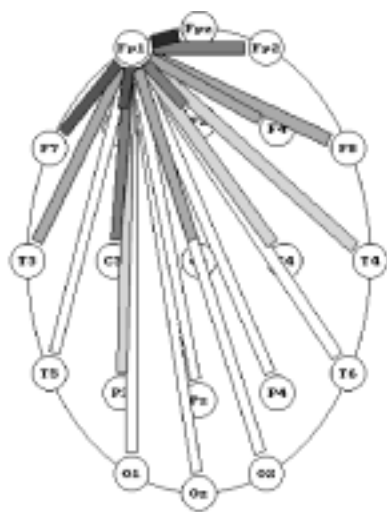
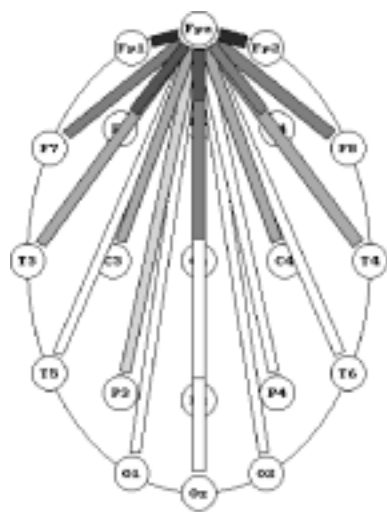
stan czuwania, oczy zamknięte

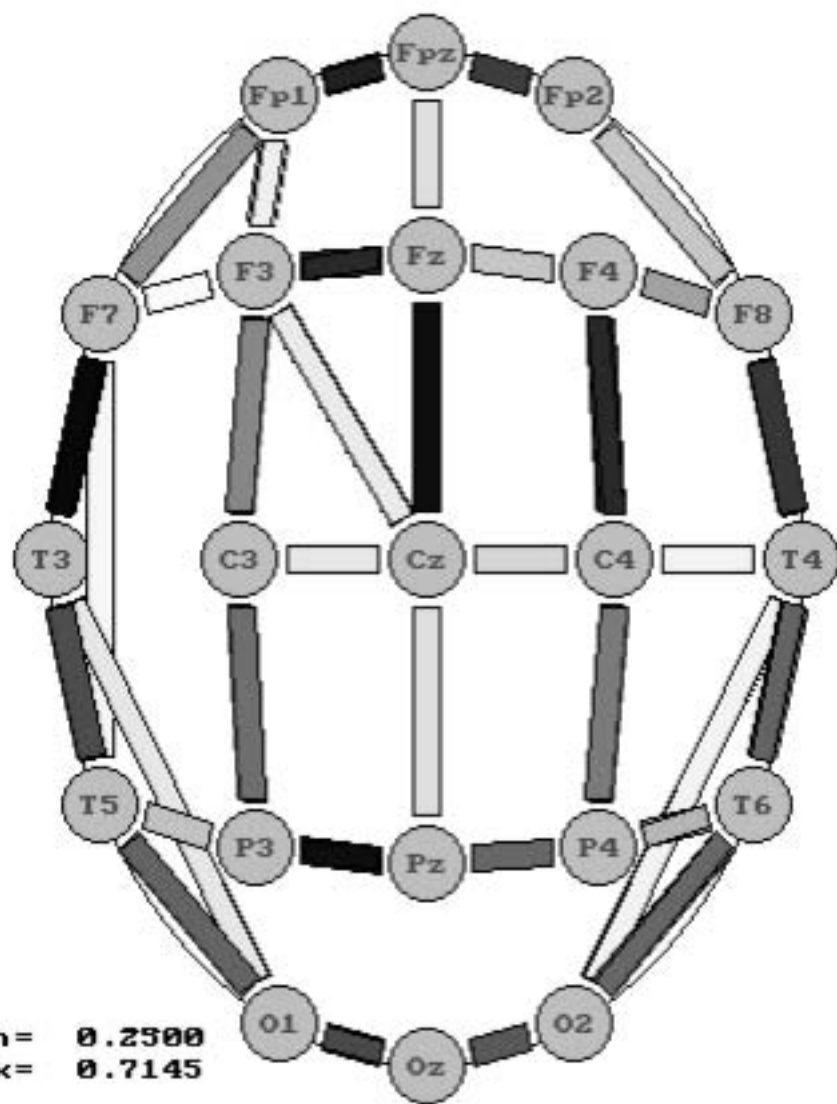


Zagadnienia pokrewne

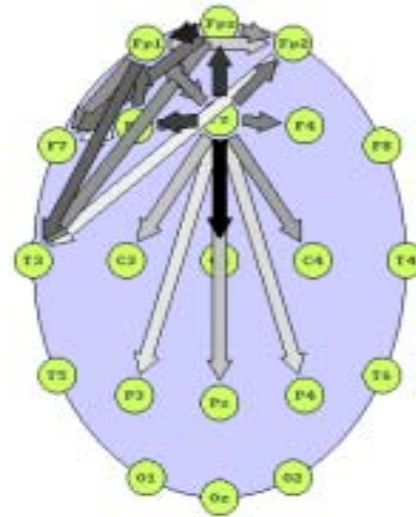
Przesyłanie wzajemne



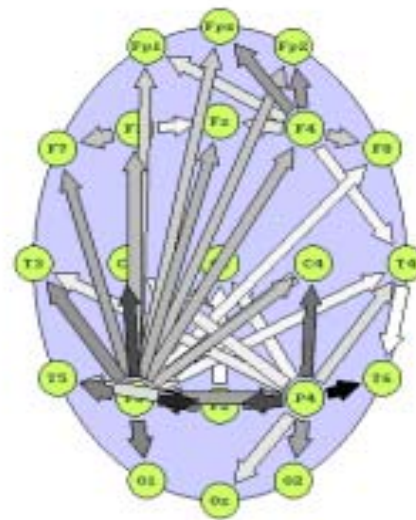
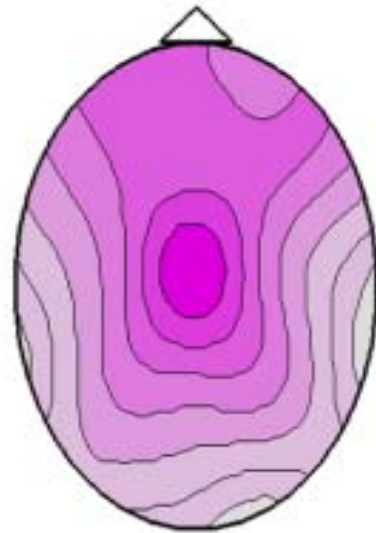




Directions of EEG activity propagation during sleep



sleep stage 3



wakefulness

Podsumowanie



Liczenie kierunku przepływu między parami kanałów może prowadzić do błędnych wyników.

Należy stosować metody uwzględniające wszystkie kanały naraz.

Wielokanałowy model autoregresyjny (**MVAR**) spełnia te wymagania — wszystkie kanały są przetwarzane jednocześnie.

Funkcja DTF bazująca na modelu MVAR pozwala prawidłowo wyznaczyć kierunki przepływu sygnału w funkcji częstości.

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