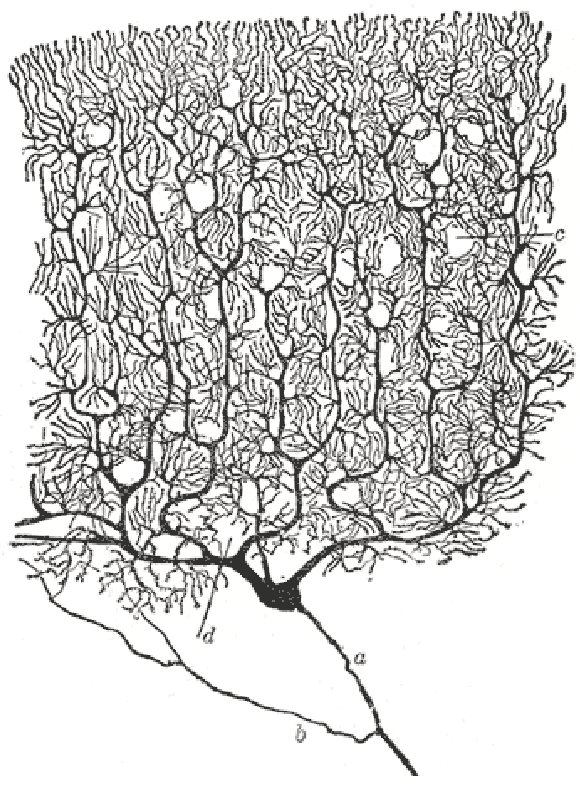
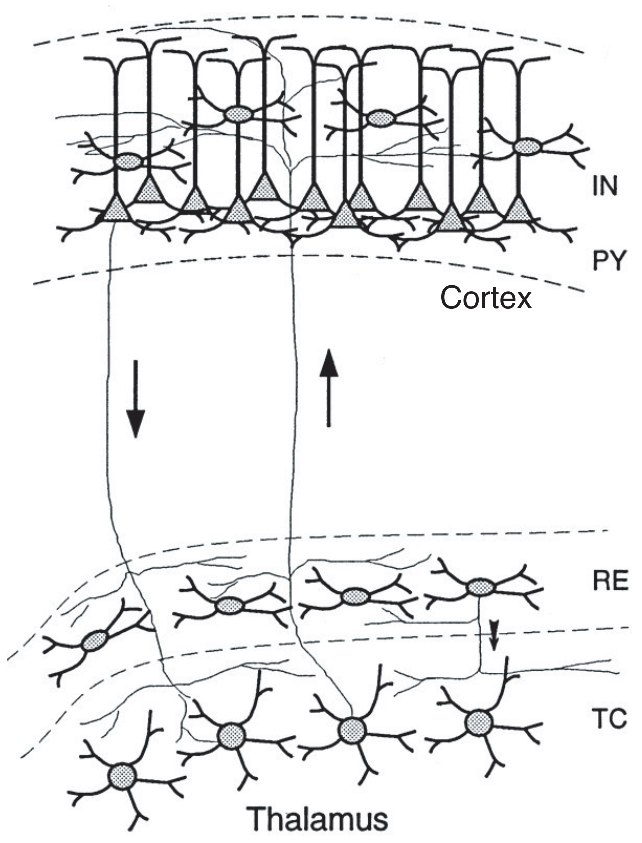
Neural field models



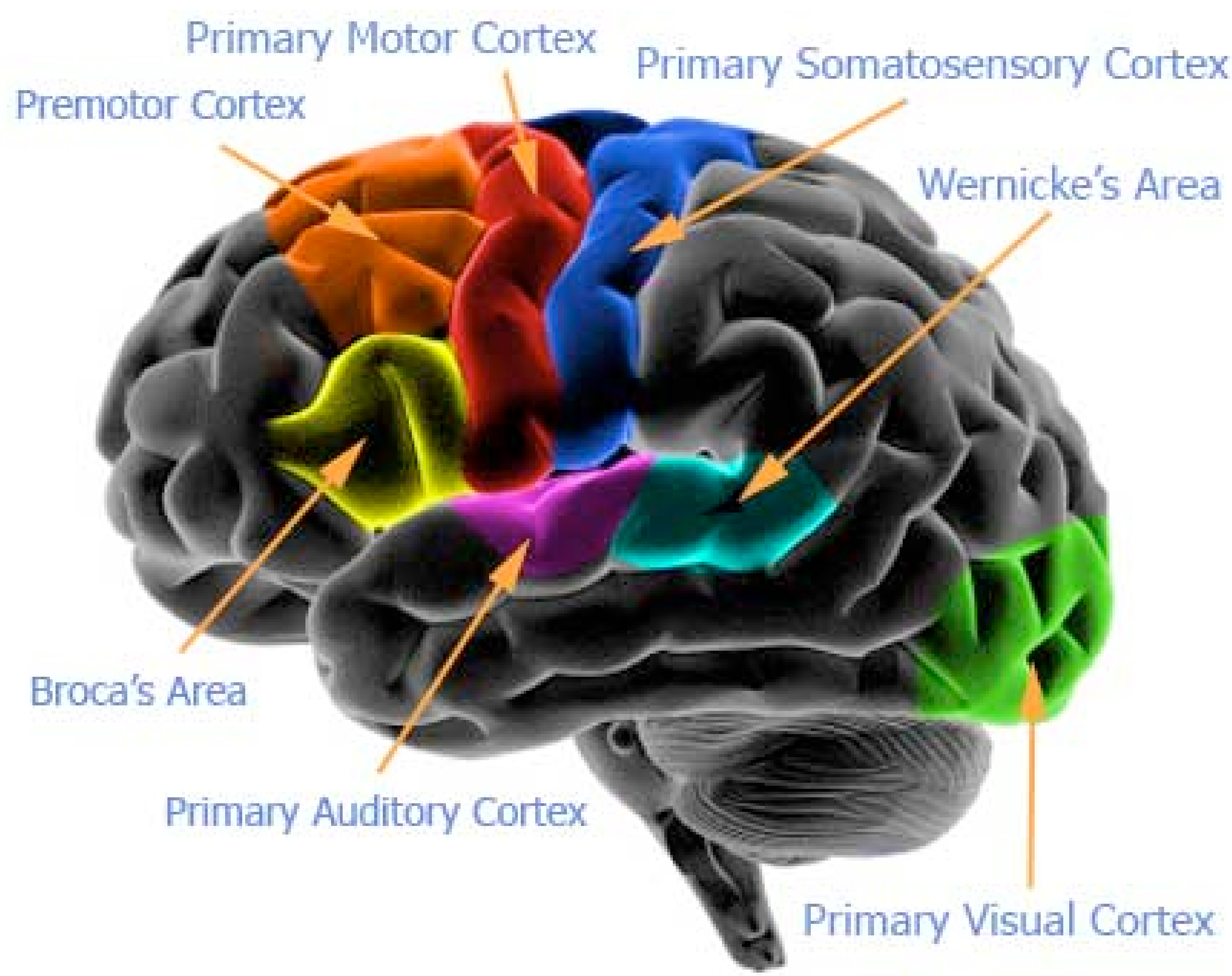
Steve
Coombes

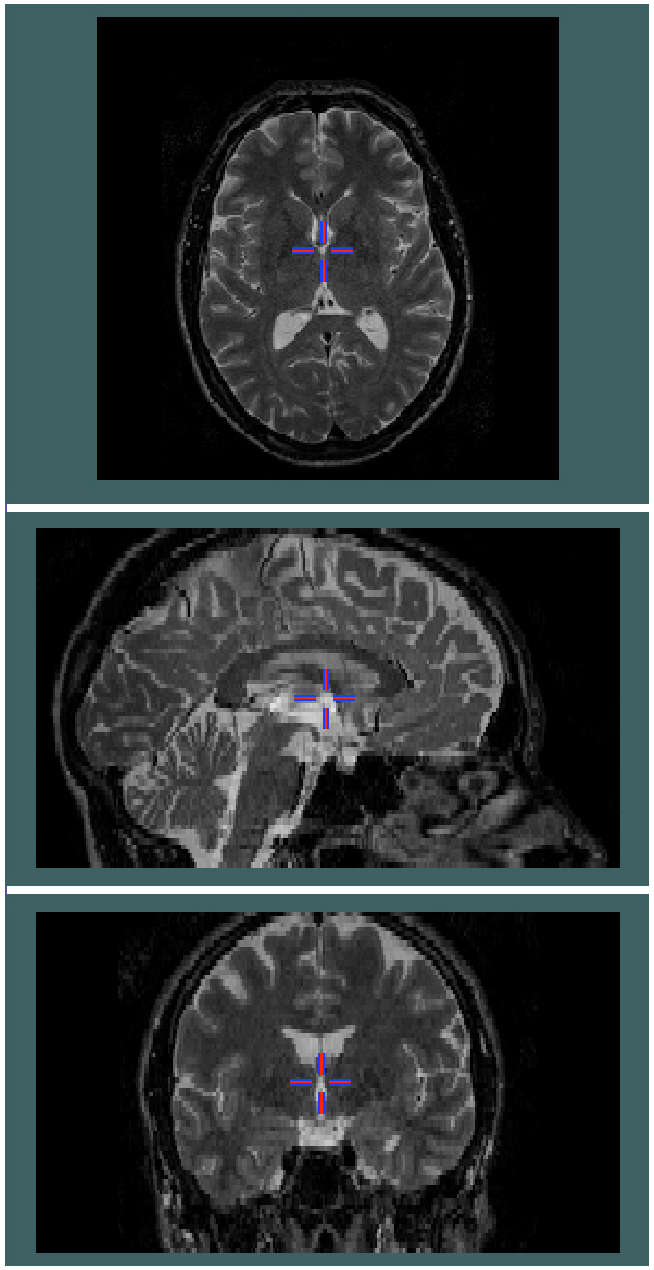


The University of Nottingham

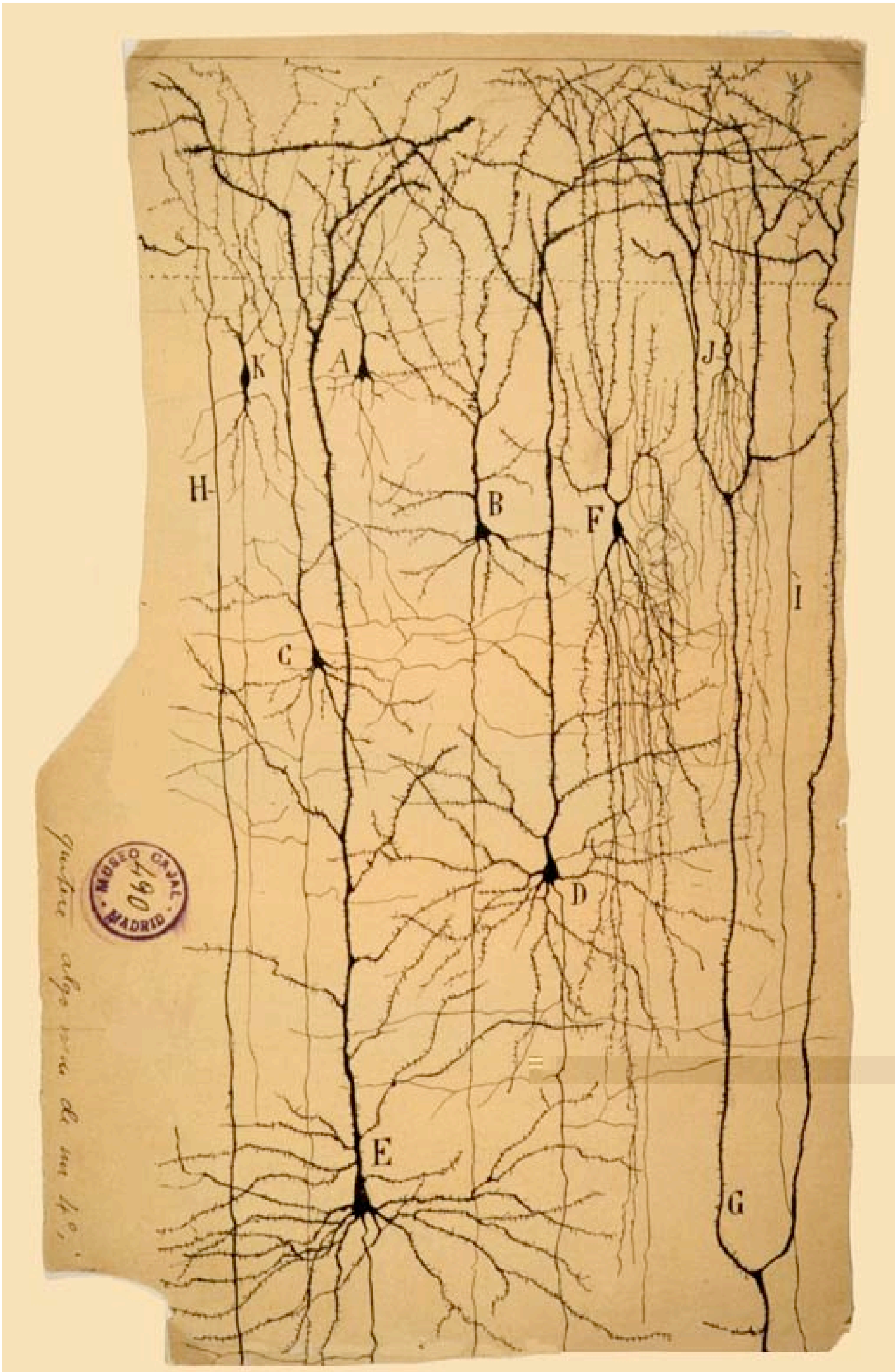
School of Mathematical Sciences

Brain and Cortex

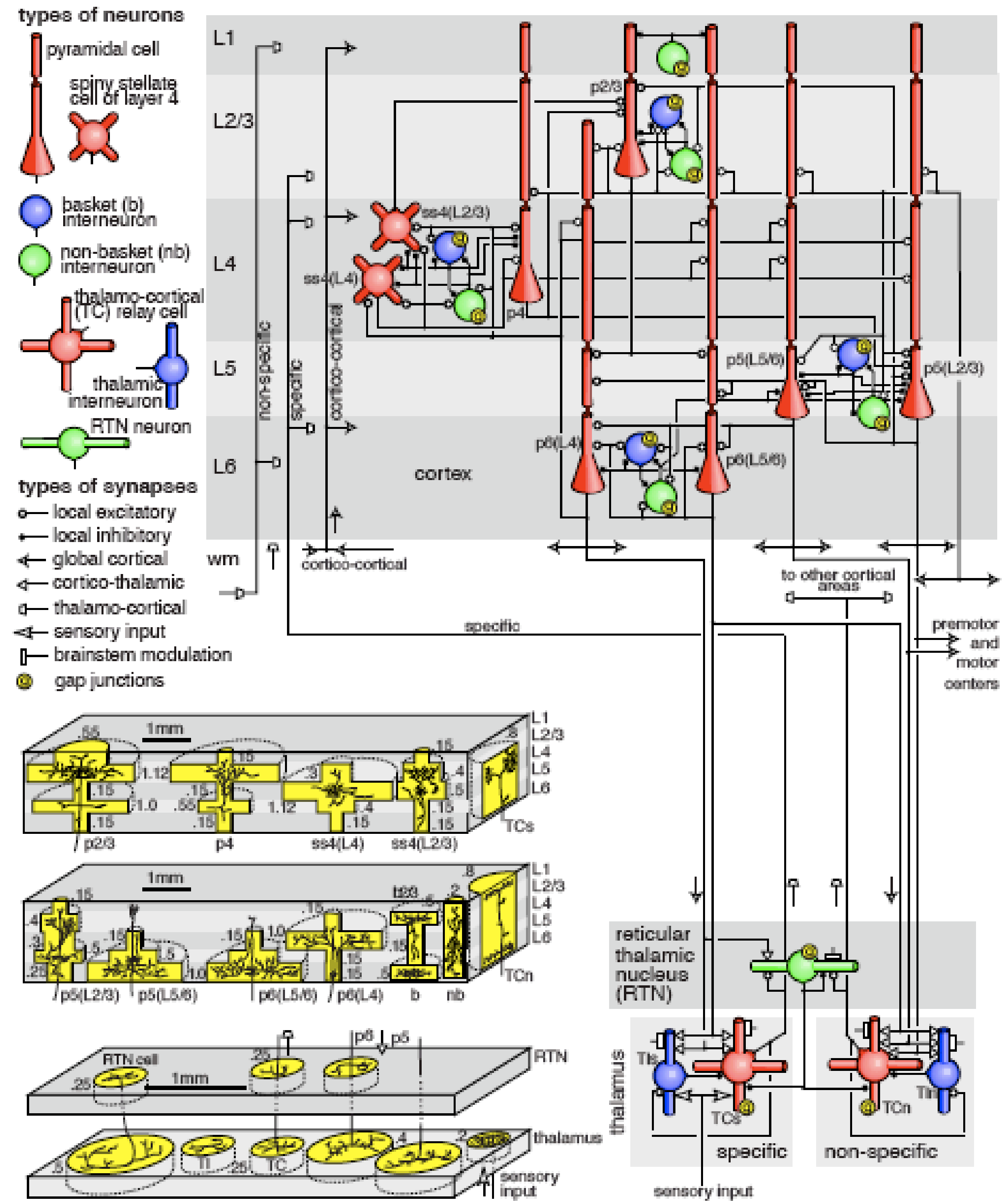




Principal cells and interneurons

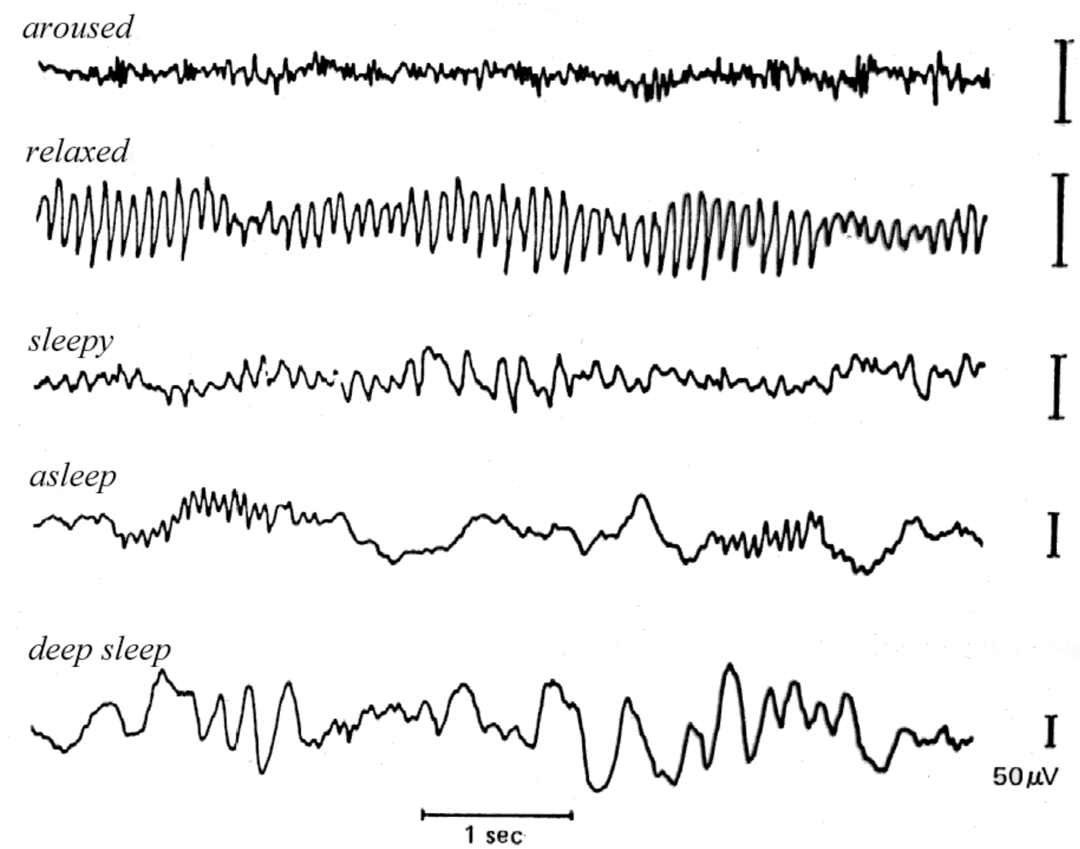


Santiago Ramón y Cajal
1900

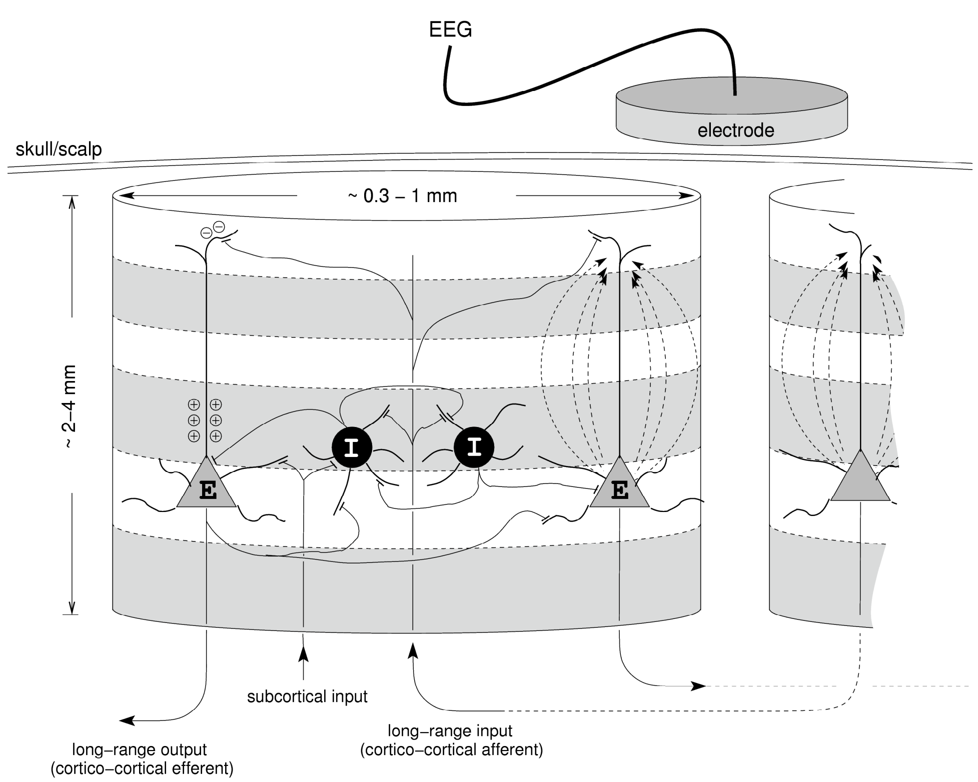


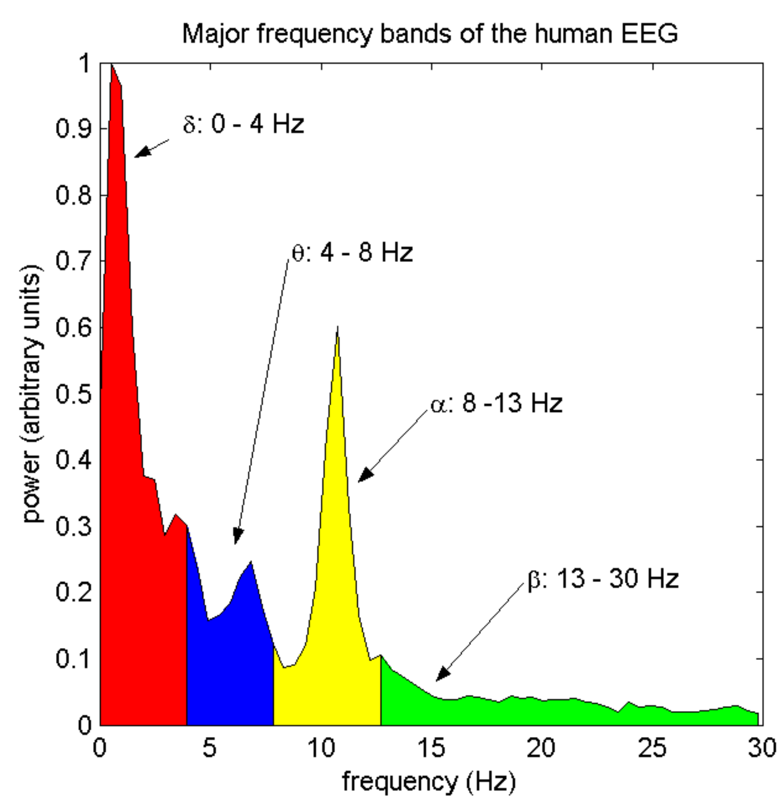
Eugene Izhikevich
2008

Electroencephalogram (EEG) power spectrum



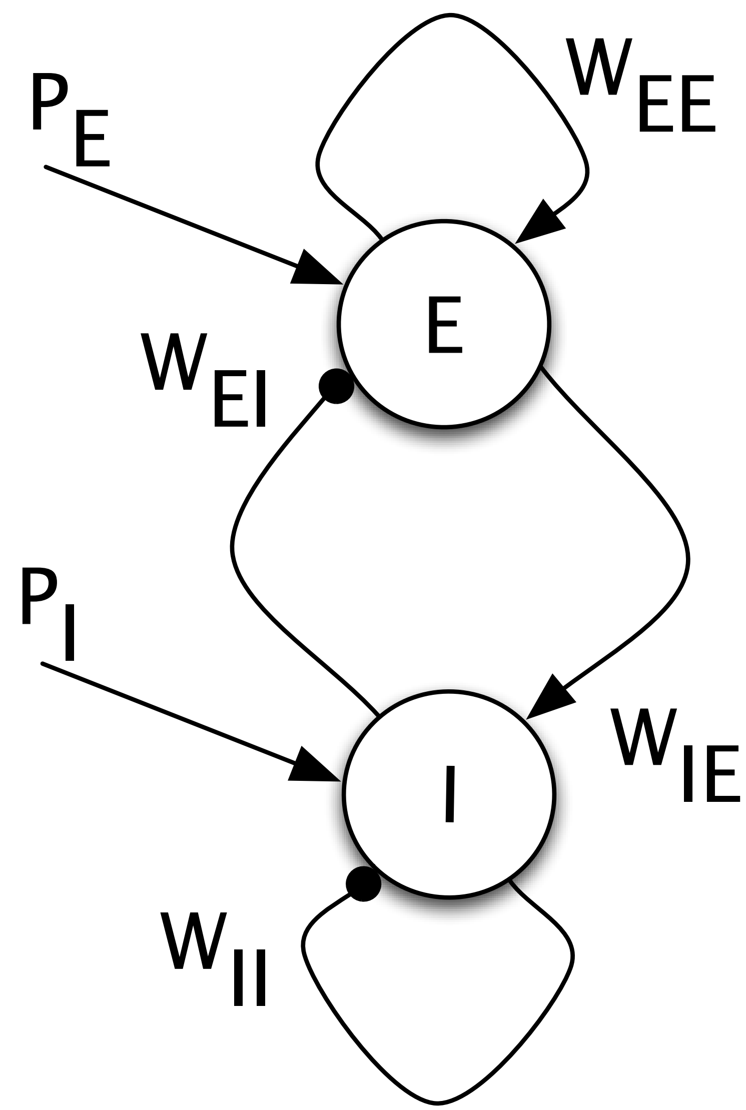
α



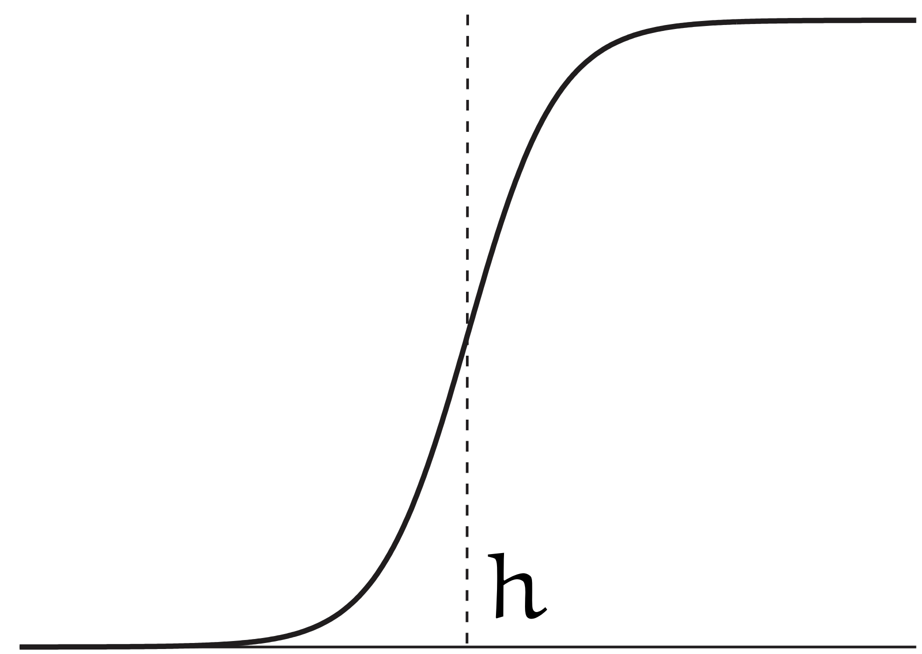


EEG records the activity of ~ 10^6 pyramidal neurons.

Population model



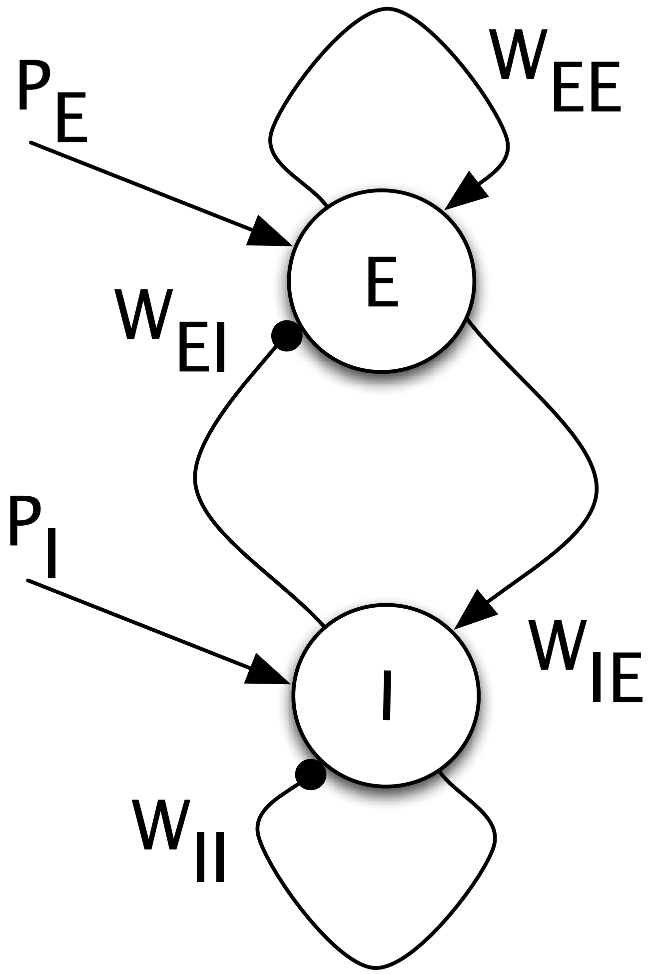
Firing rate activity $f(E)$

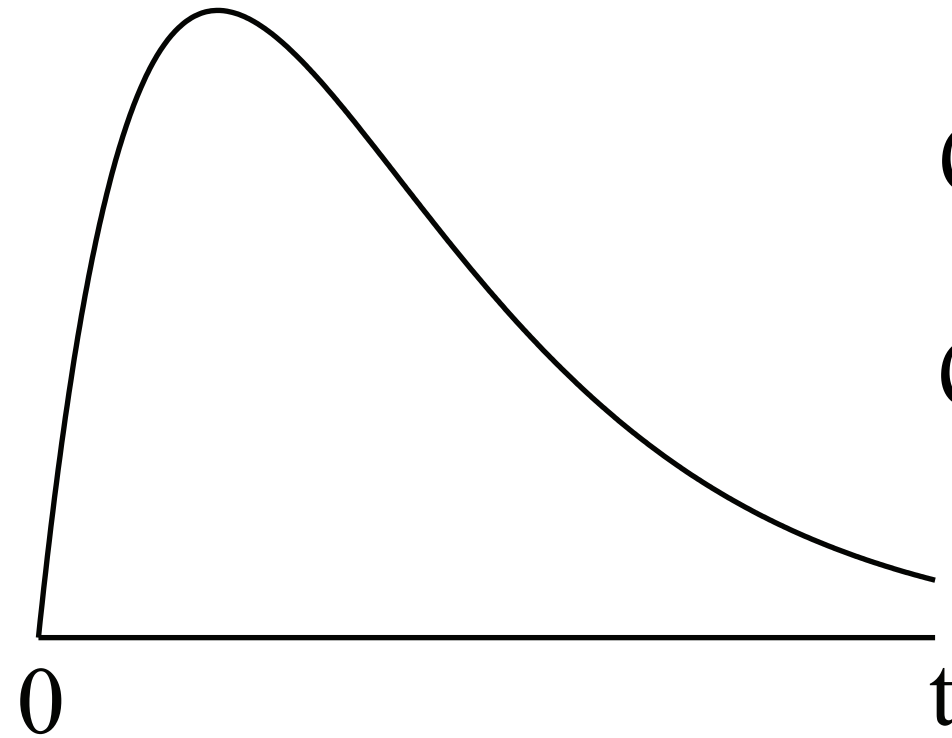


Firing rate activity $f(I)$

$$\dot{E} = -\frac{E}{\tau_E} + W_{EE} g_{EE}(A^+ - E) + W_{EI} g_{EI}(A^- - E) + P_E$$

$$\dot{I} = -\frac{I}{\tau_I} + W_{II} g_{II}(A^- - I) + W_{IE} g_{IE}(A^+ - I) + P_I$$





$$\eta(t) = \alpha^2 t e^{-\alpha t}$$

$$Q\eta = \delta$$

$$Q = \left(1 + \frac{1}{\alpha}\frac{d}{dt}\right)^2$$

$$Qg_{jE} = f(E) \qquad Qg_{jI} = f(I)$$

Steady state approximation

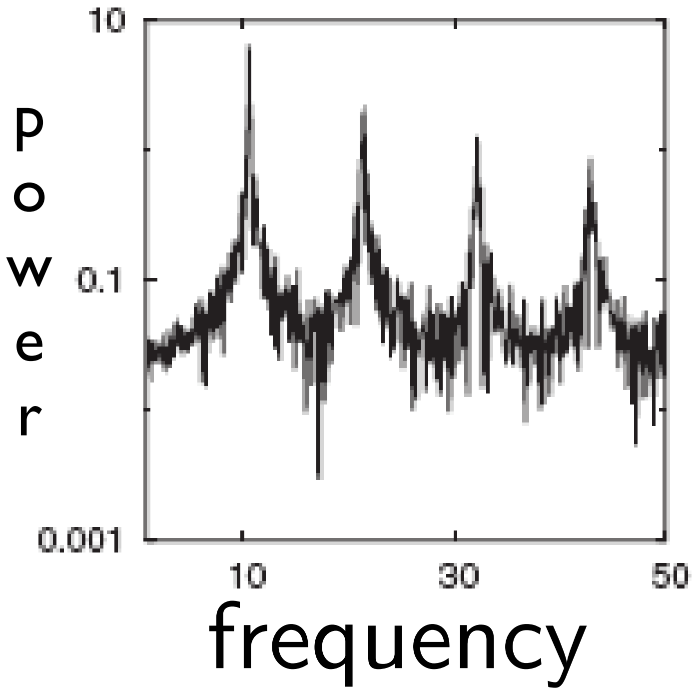
$$E = E(g_{EE}, g_{EI}) \qquad I = I(g_{II}, g_{IE})$$

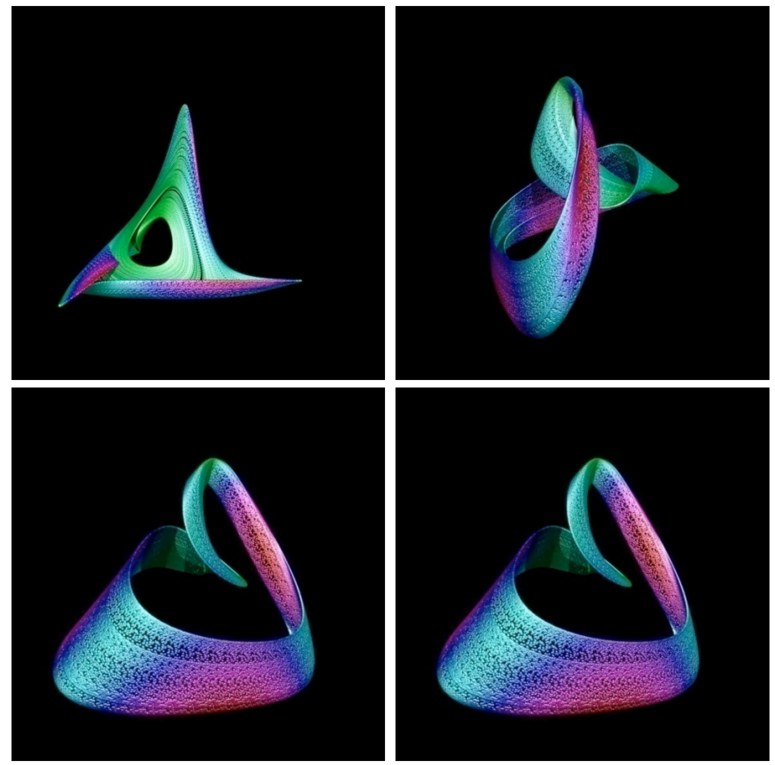
$$Qg = f$$

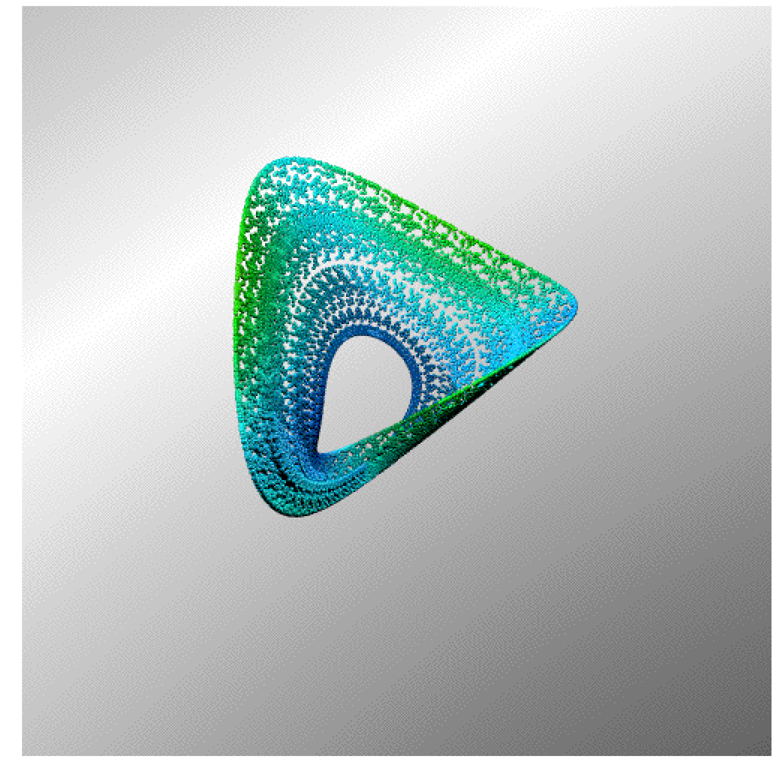
$$f = f(\{g\})$$

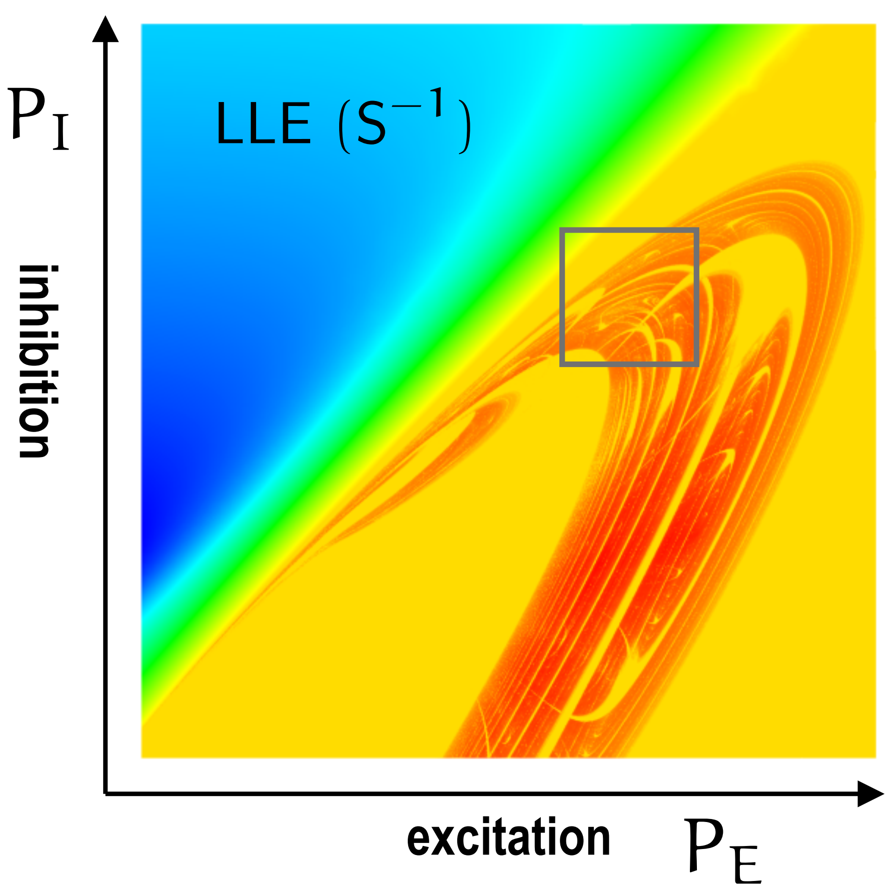
$$g = \eta * f$$

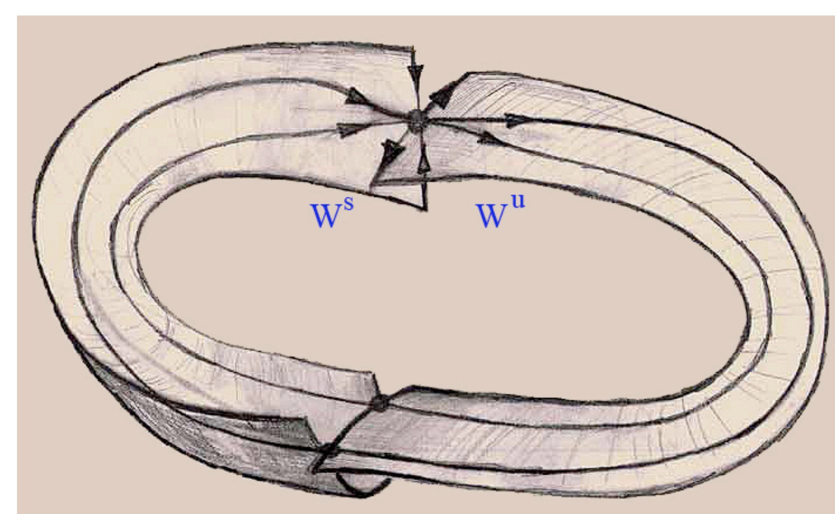
Alphoid chaos (10 D)











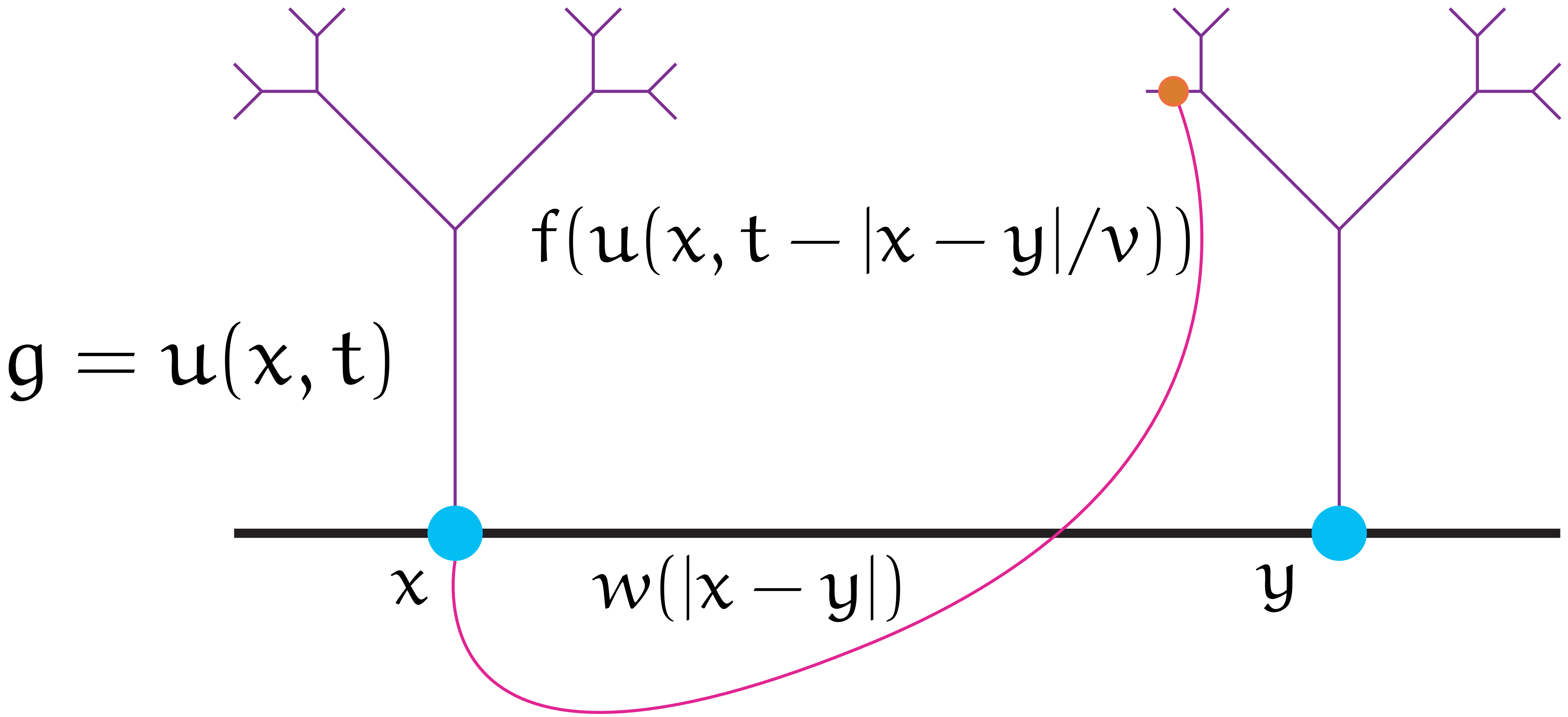
Shilnikov saddle-node route to chaos

van Veen and Liley, PRL, **97**, 208101 (2006)

Spatially extended models

$$g = w \otimes \eta * f$$

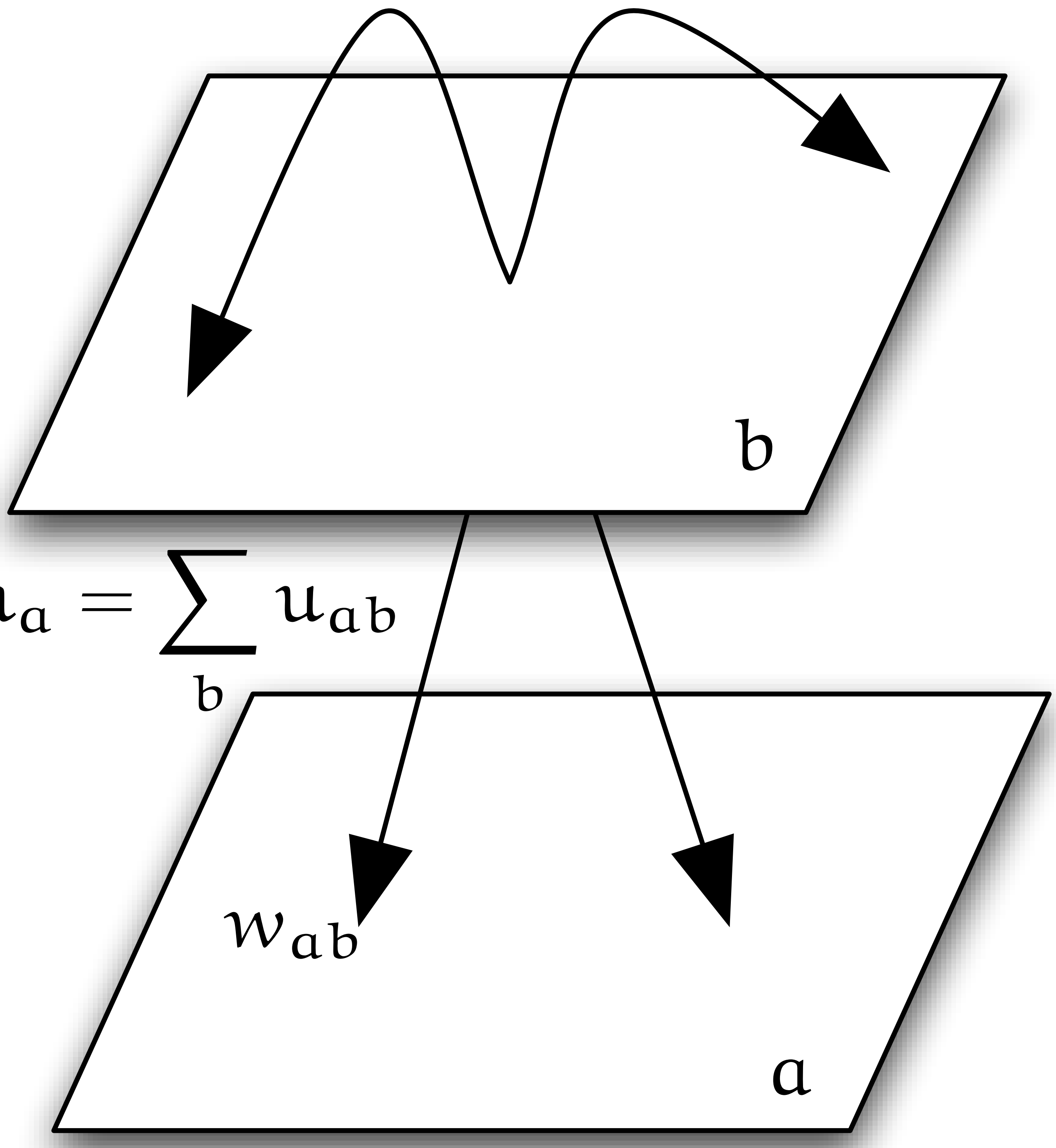
Simplest neural field model: Wilson-Cowan ('72), Amari ('77)



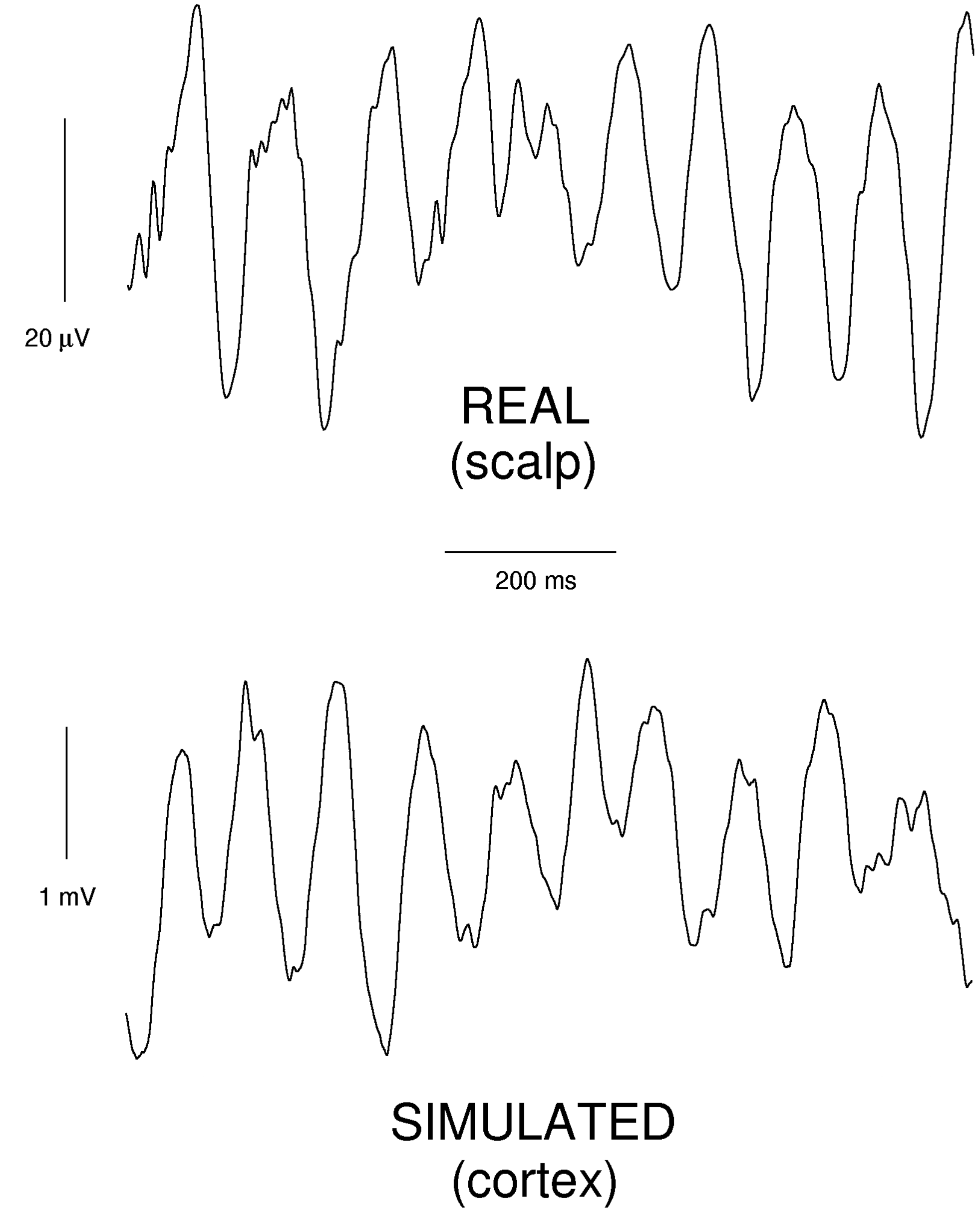
$$u(x,t) = \int_{-\infty}^{\infty} dy\ w(y) \int_0^{\infty} ds\ \eta(s)\ f\left(u(x-y, t-s-|y|/v)\right)$$

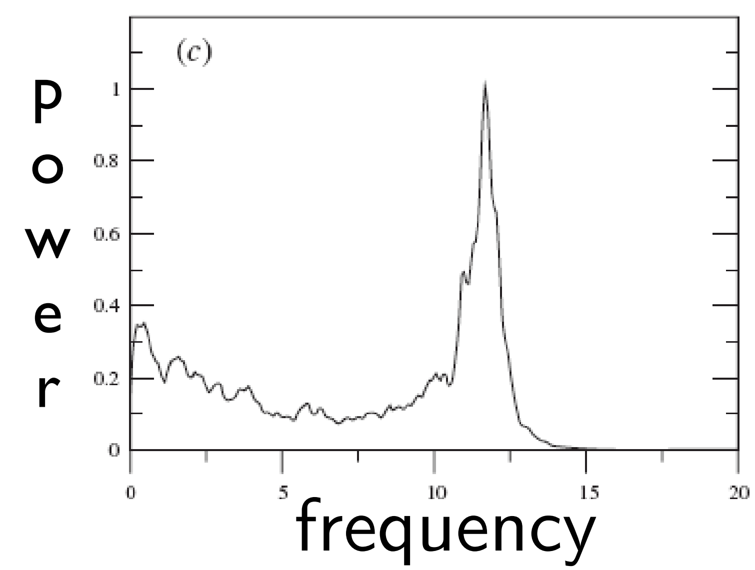
2D layers

$$u_{ab} = \eta_{ab} * \psi_{ab}$$



$$h_a = \sum_b u_{ab}$$

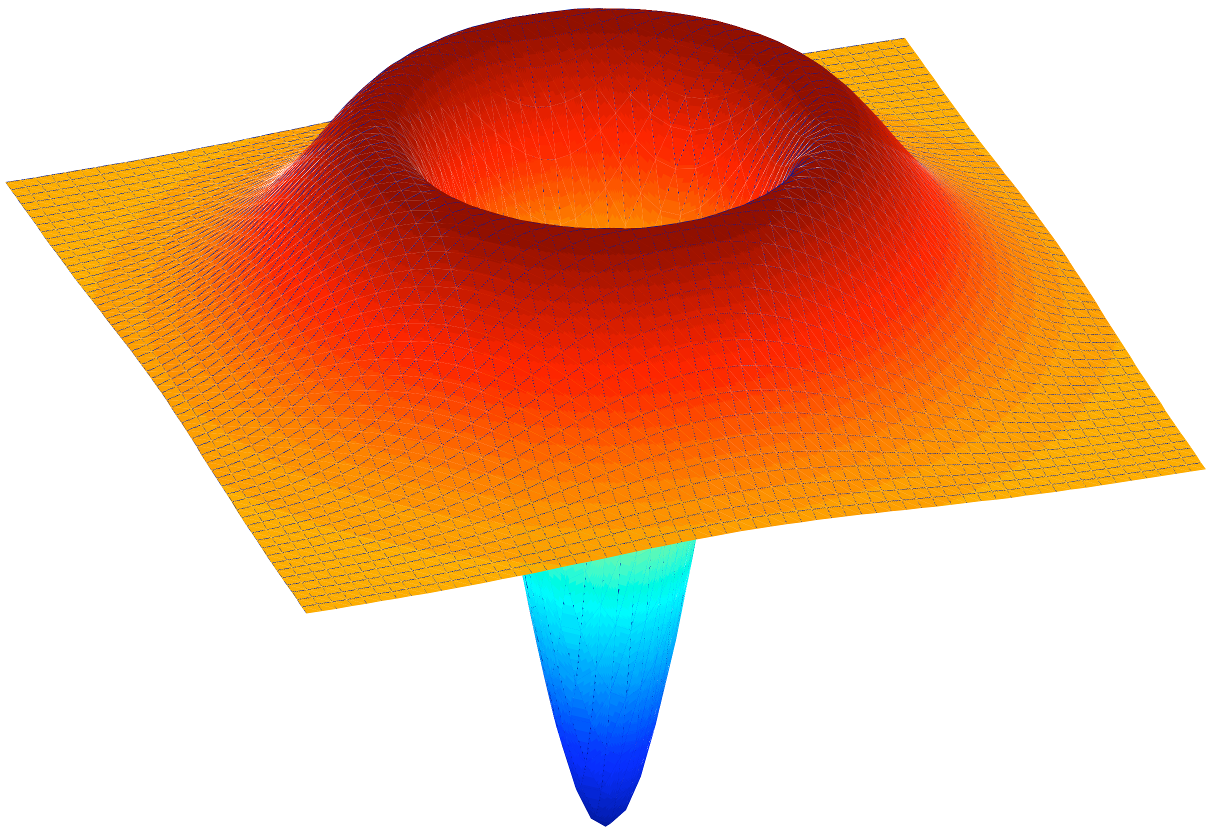




$$\psi_{ab}(\mathbf{r}, t) = \int_{\mathbb{R}^2} d\mathbf{r}' \; w_{ab}(\mathbf{r}, \mathbf{r}') f_b \circ h_b \left(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'|/v_{ab}\right)$$

Turing instability analysis

E layer and I layer



$$e^{i\mathbf{k}\cdot\mathbf{r}}e^{\lambda t}$$

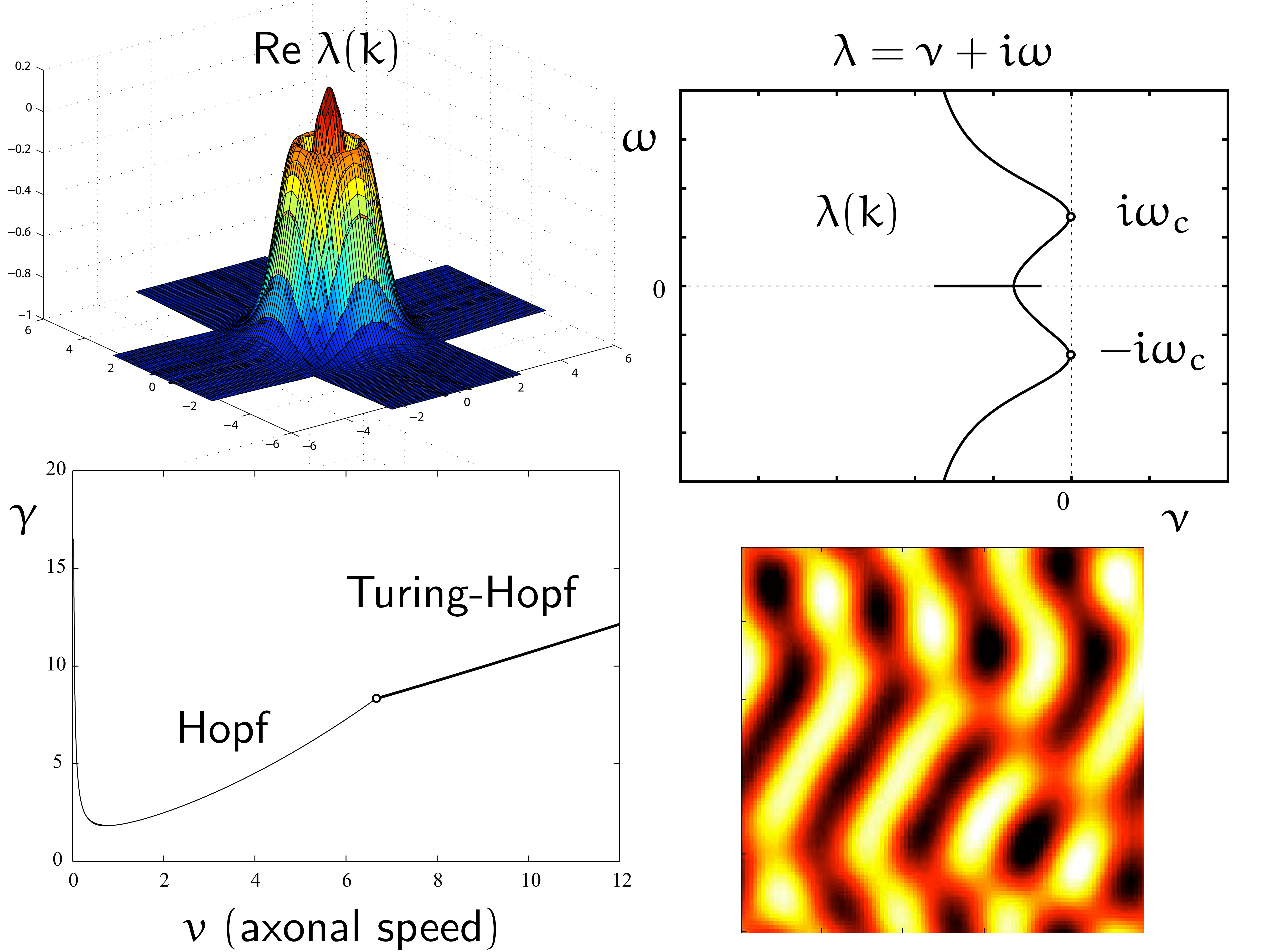
Continuous spectrum

$$\det\left(\mathcal{D}(k,\lambda) - I\right) = 0$$

$$[\mathcal{D}(k,\lambda)]_{ab} = \widetilde{\eta}_{ab}(\lambda) G_{ab}(k, -i\lambda)\gamma_b$$

$\widetilde{\eta} = \text{LT } \eta$ $\qquad$ $G = \text{FLT } w(r)\delta(t - r/v)$ $\qquad$ $\gamma = f'(ss)$

S Coombes et al., PRE, **76**, 05190 (2007)


Re λ(k)
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
λ = ν + iω
ω
λ(k)
iω_c
0
−iω_c
0
ν
20
γ
15
Turing-Hopf
10
Hopf
5
0
0
2
4
6
8
10
12
ν (axonal speed)

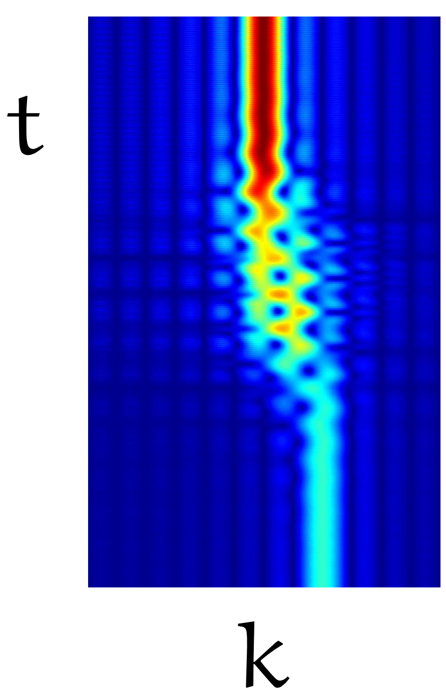
Amplitude Equations (one D)

Coupled mean-field Ginzburg–Landau equations describing a Turing–Hopf bifurcation with modulation group velocity of $O(1)$.

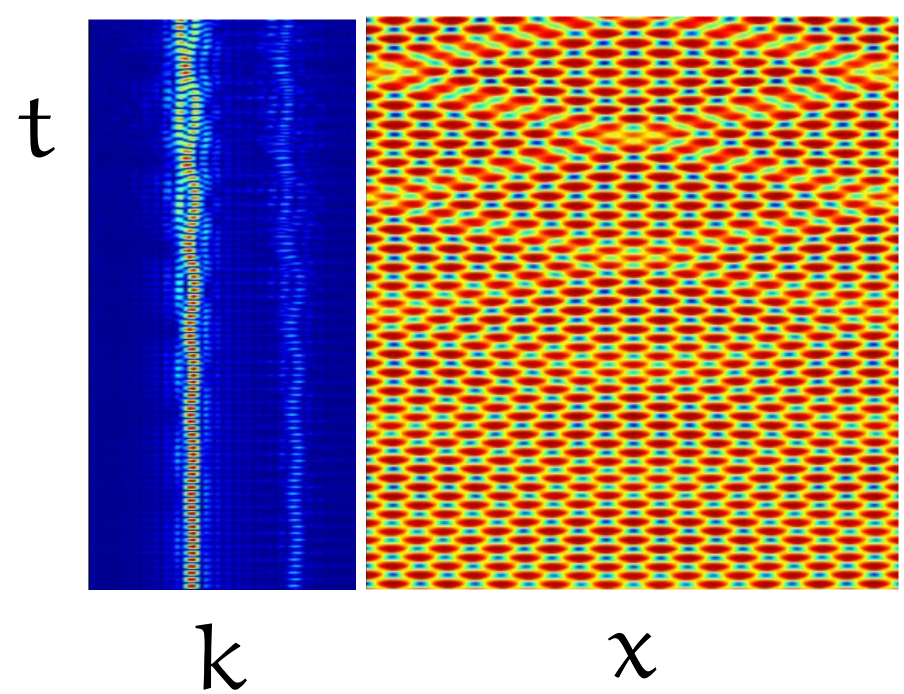
$$\frac{\partial A_1}{\partial \tau} = A_1(a + b|A_1|^2 + c\langle |A_2|^2\rangle) + d\frac{\partial^2 A_1}{\partial \xi_+^2}$$

$$\frac{\partial A_2}{\partial \tau} = A_2(a + b|A_2|^2 + c\langle |A_1|^2\rangle) + d\frac{\partial^2 A_2}{\partial \xi_-^2}$$

Benjamin–Feir (BF)



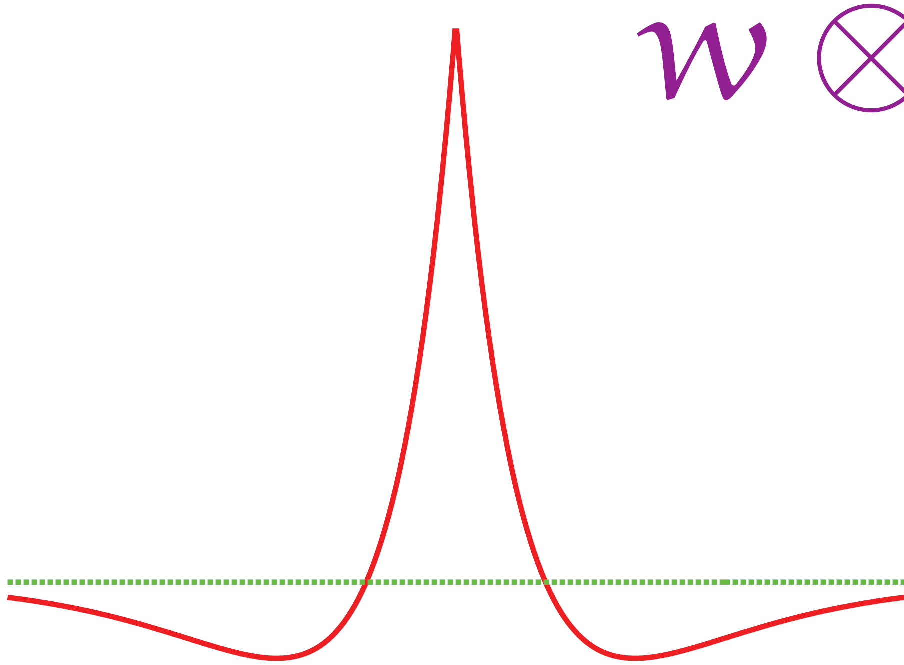
BF-Eckhaus instability



Coefficients in terms of integral transforms of w and η .

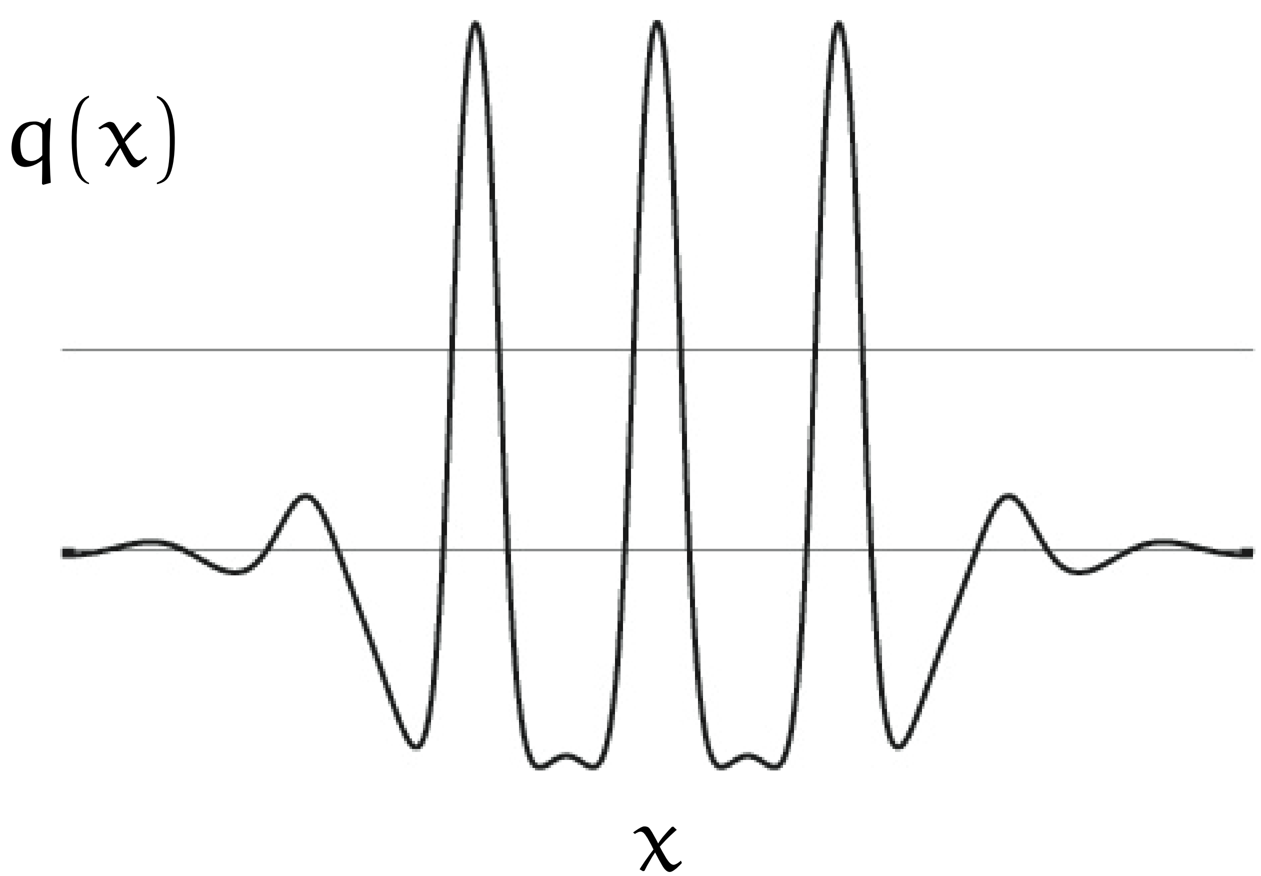
Time independent localised solutions

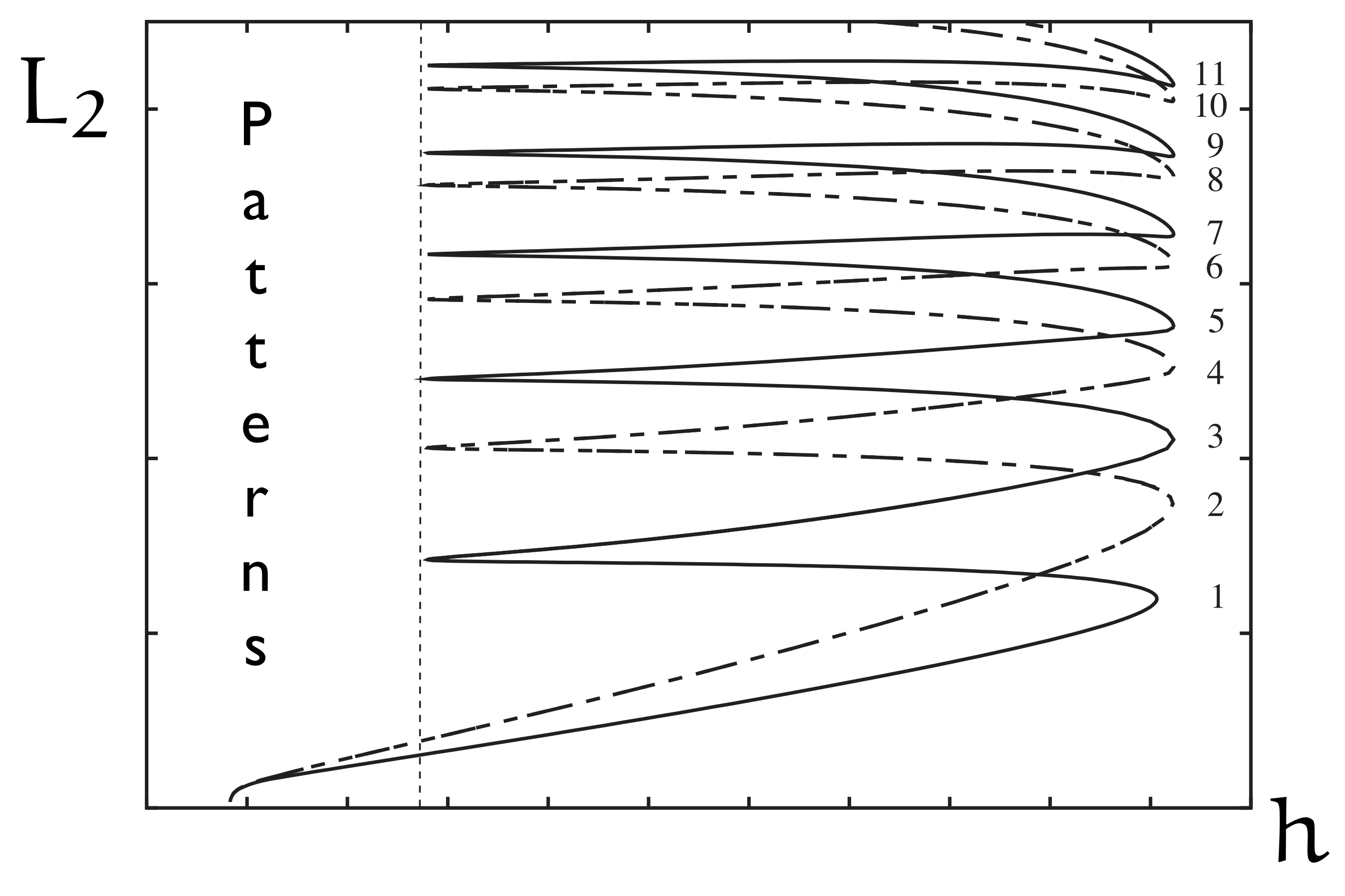
$$w \otimes \eta * f \rightarrow w \otimes f$$



$$q(x) = \int_{\mathbb{R}} dy \; w(x-y) f \circ q(y)$$

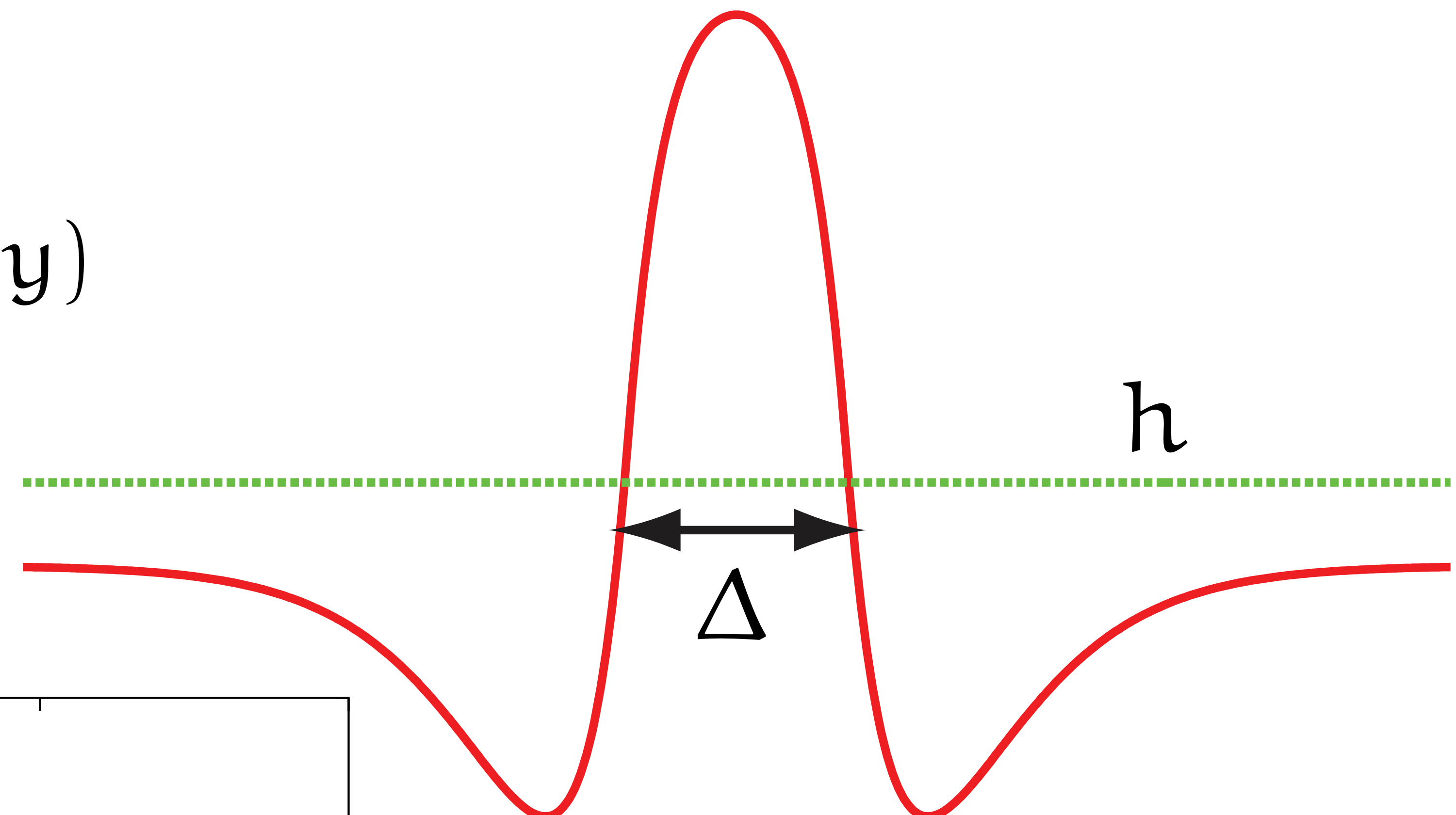
$$w(x) = (1 - |x|)e^{-|x|}$$

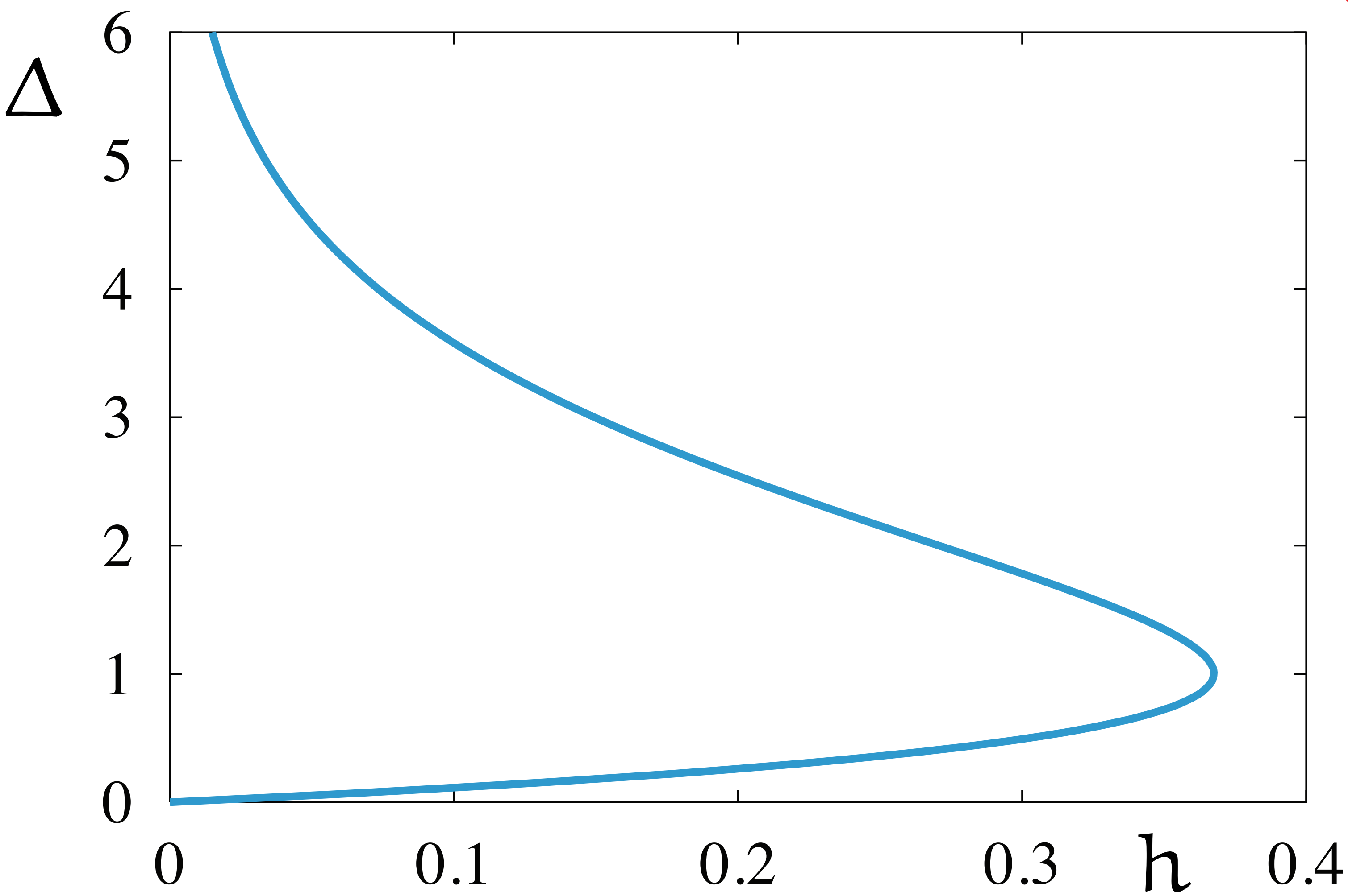




Exact result for 1-bump: $f(u) = H(u - h)$

$$q(x) = \int_0^{\Delta} dy\; w(x - y)$$





$$q(0) = h = q(\Delta)$$

$$\Delta e^{-\Delta} = h$$

working memory

Stability

Examine eigenspectrum of the linearization about a solu

Solutions of form $u(x)e^{\lambda t}$ satisfy $\mathcal{L}u(x) = u(x)$

$$\mathcal{L}u(x) = \tilde{\eta}(\lambda)\int_{-\infty}^{\infty} dy\; w(x-y)f'(q(y)-h)u(y)$$

For Heaviside firing rate

$$f'(q(x)) = \frac{\delta(x)}{|q'(0)|} + \frac{\delta(x-\Delta)}{|q'(\Delta)|}$$

so

$$u(x) = \frac{\tilde{\eta}(\lambda)}{|w(0)-w(\Delta)|}[w(x)u(0) + w(x-\Delta)u(\Delta)]$$

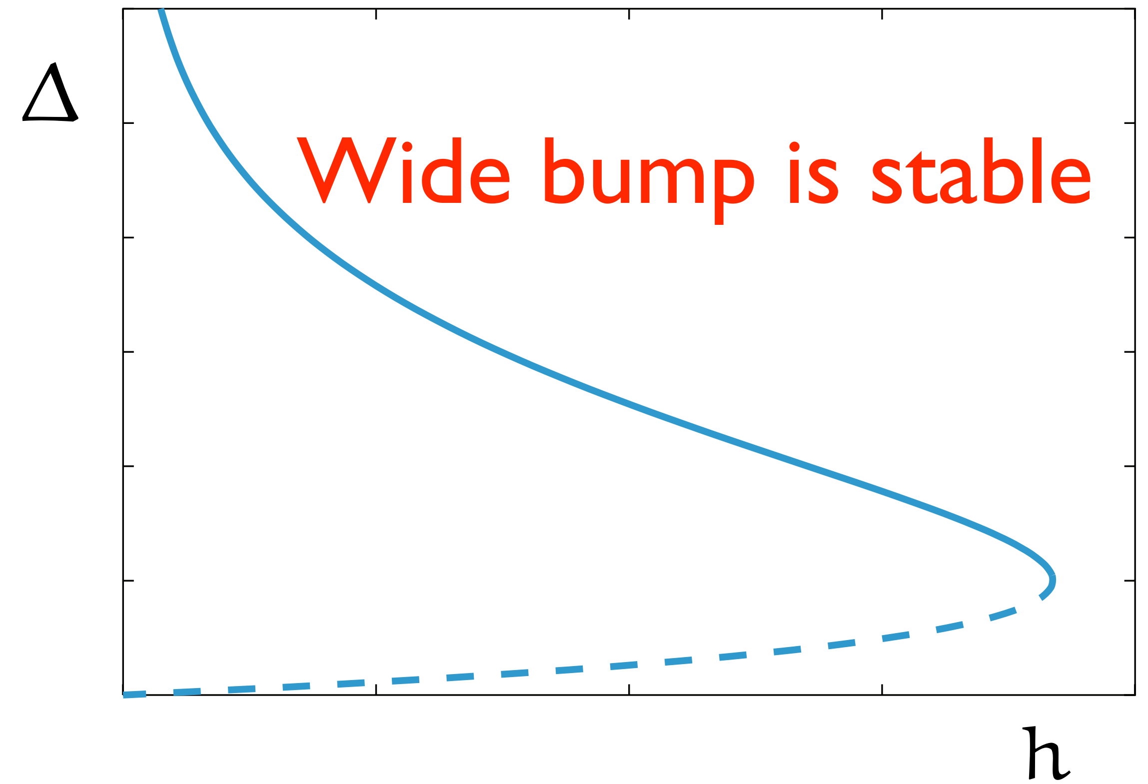
System of linear equations for perturbations at threshold

$$\begin{bmatrix} u(0) \\ u(\Delta) \end{bmatrix} = \mathcal{A}(\lambda) \begin{bmatrix} u(0) \\ u(\Delta) \end{bmatrix}, \qquad \mathcal{A}(\lambda) = \frac{\widetilde{\eta}(\lambda)}{|w(0) - w(\Delta)|} \begin{bmatrix} w(0) & w(\Delta) \\ w(\Delta) & w(0) \end{bmatrix}$$

Non trivial solution if

$$\mathcal{E}(\lambda) = \det(\mathcal{A}(\lambda) - I) = 0$$

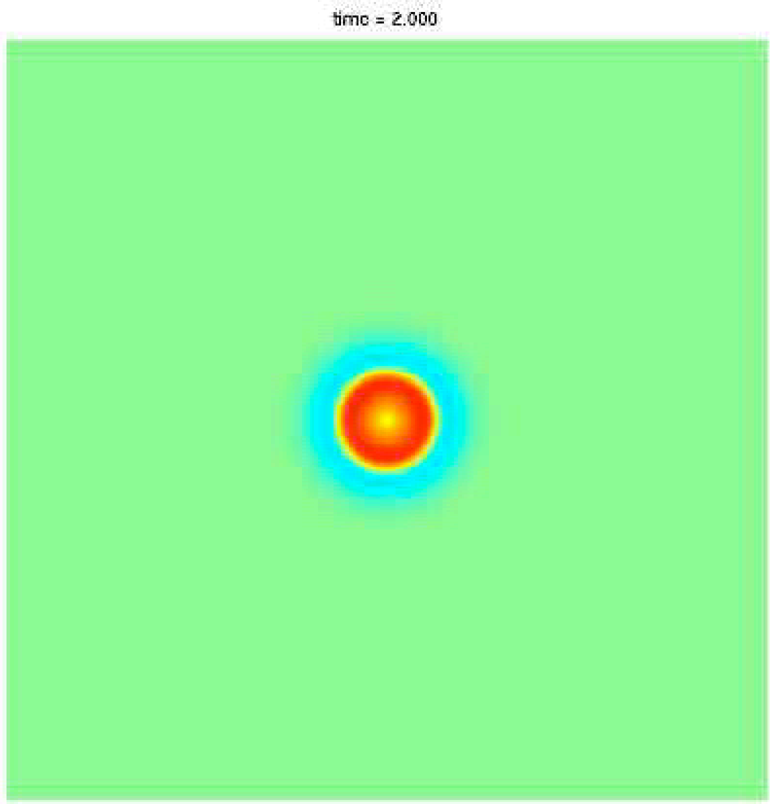
Solutions stable if Re $\lambda < 0$

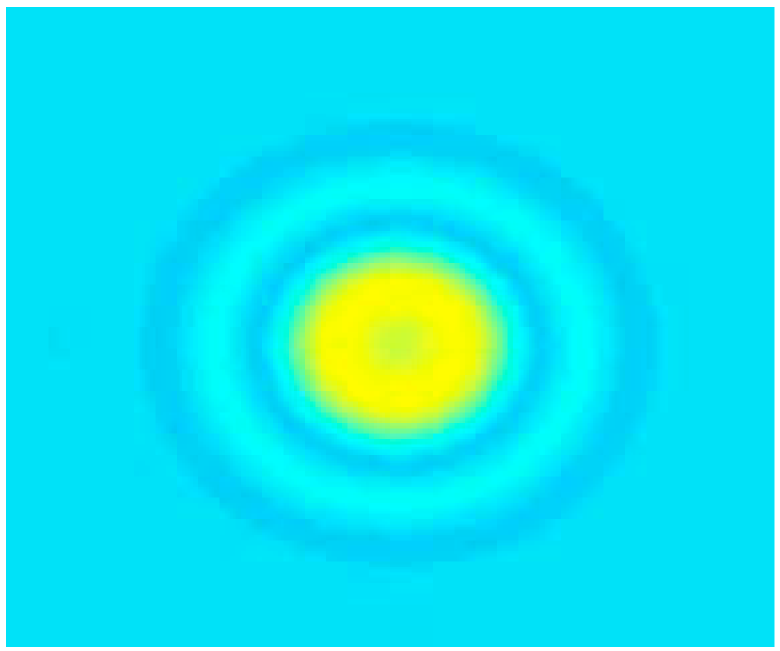


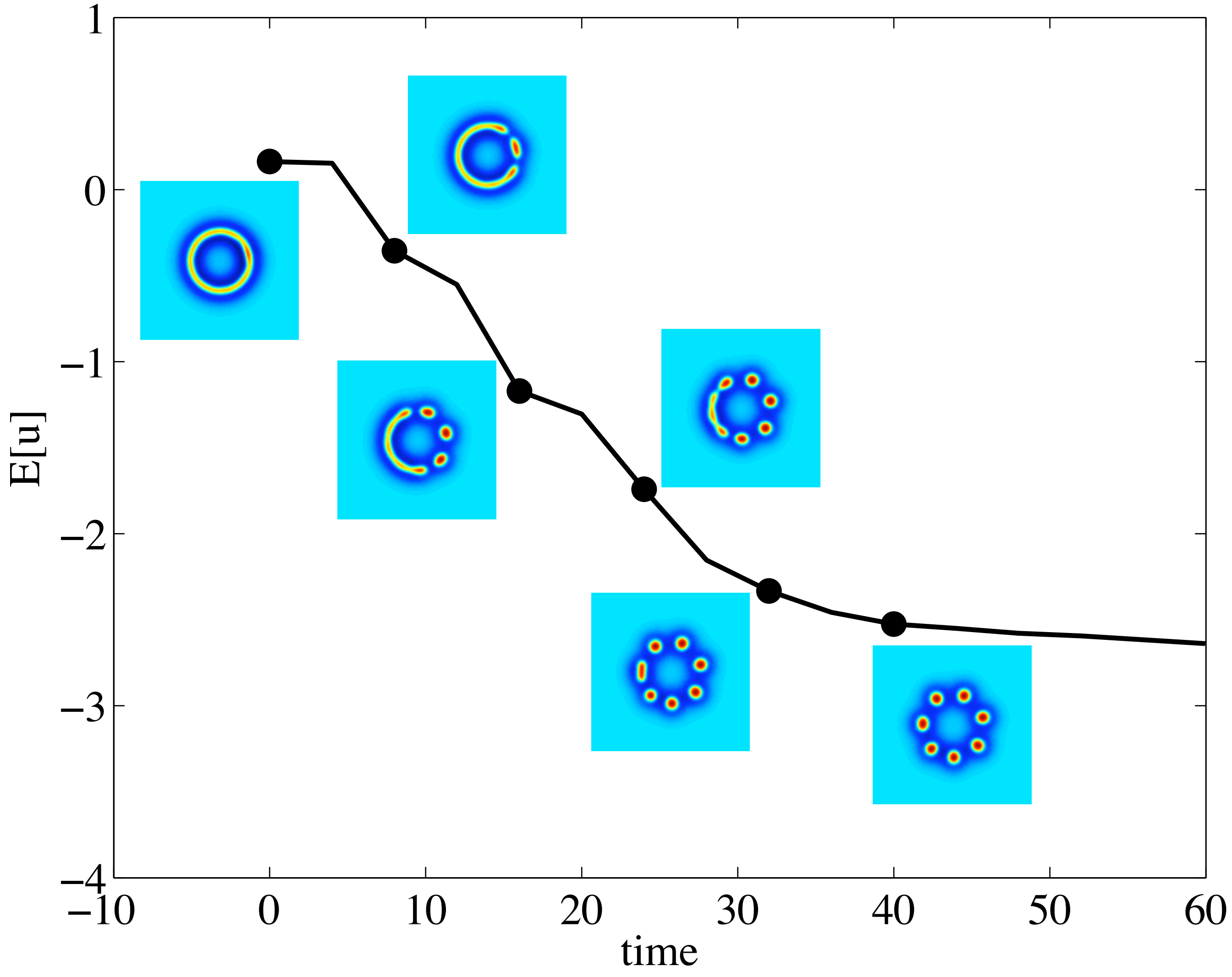
Evans function for integral neural field equation

S Coombes and M R Owen (2004) Evans functions for integral neural field equations with Heaviside firing rate function, SIAM Journal on Applied Dynamical Systems, Vol 34, 574-600.

Predictions of Evans function







M R Owen, C R Laing and S Coombes 2007 Bumps and rings in a two-dimensional neural field: splitting and rotational instabilities, New Journal of Physics, Vol 9, 378

Threshold accommodation

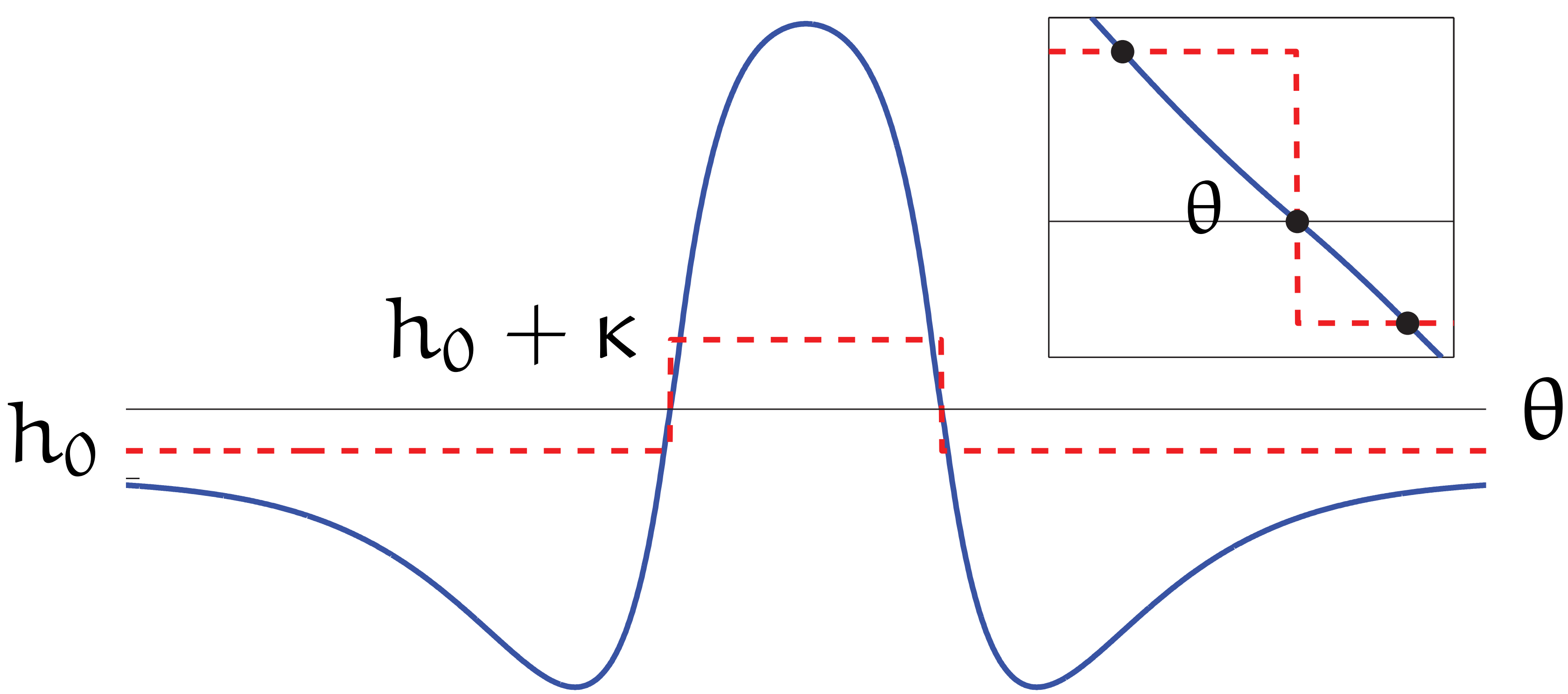
Hill (1936), "... the threshold rises when the *local potential* is maintained ... and reverts gradually to its original value when the nerve is allowed to rest."

$$\frac{\partial h}{\partial t} = -(h - h_0) + \kappa H(u - \theta)$$

One bump $(u, h) = (q(x), p(x))$

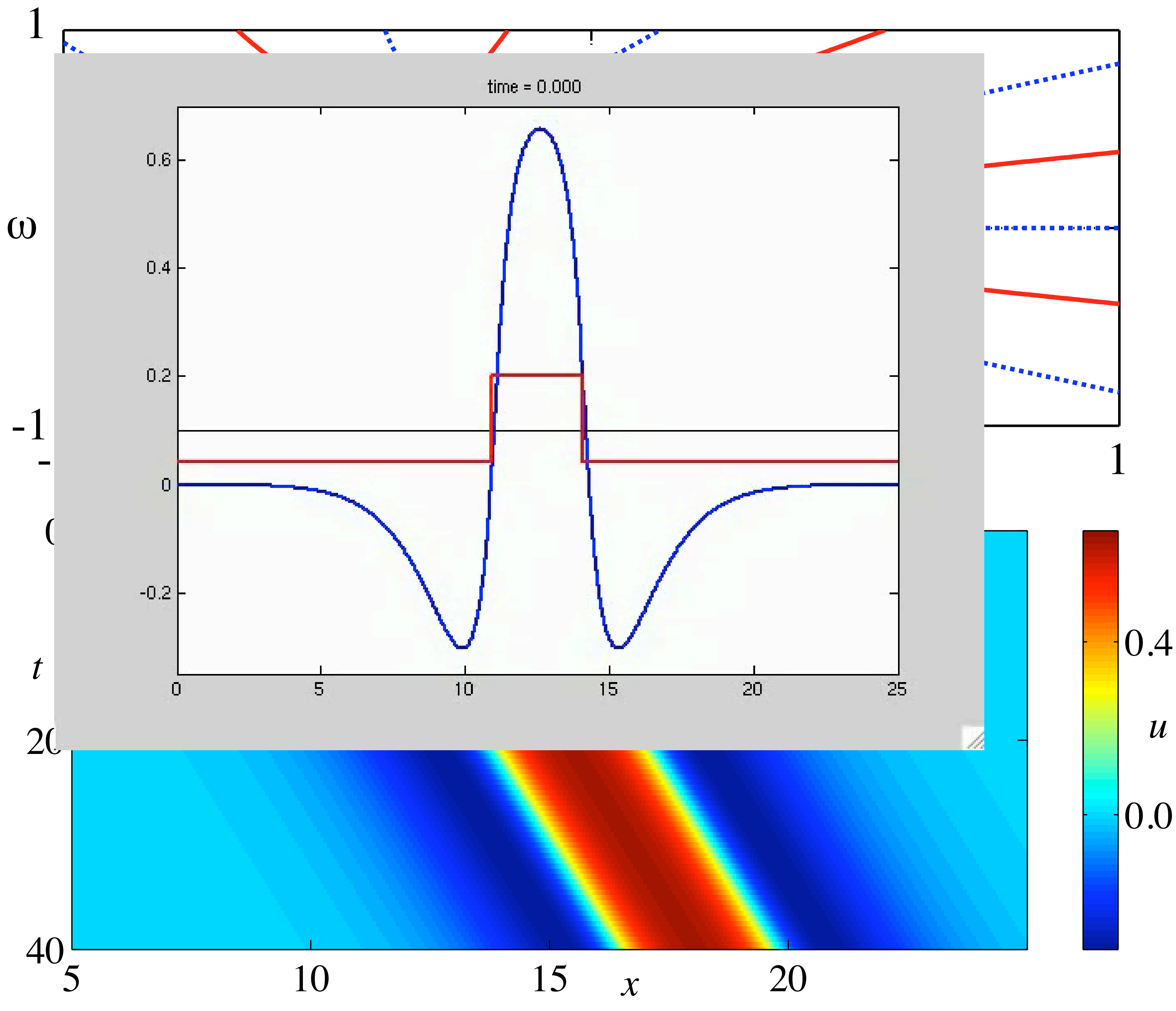
$$q = w \otimes H(q - p)$$

$$p = \begin{cases} h_0 & q < \theta \\ h_0 + \kappa & q \geq \theta \end{cases}$$



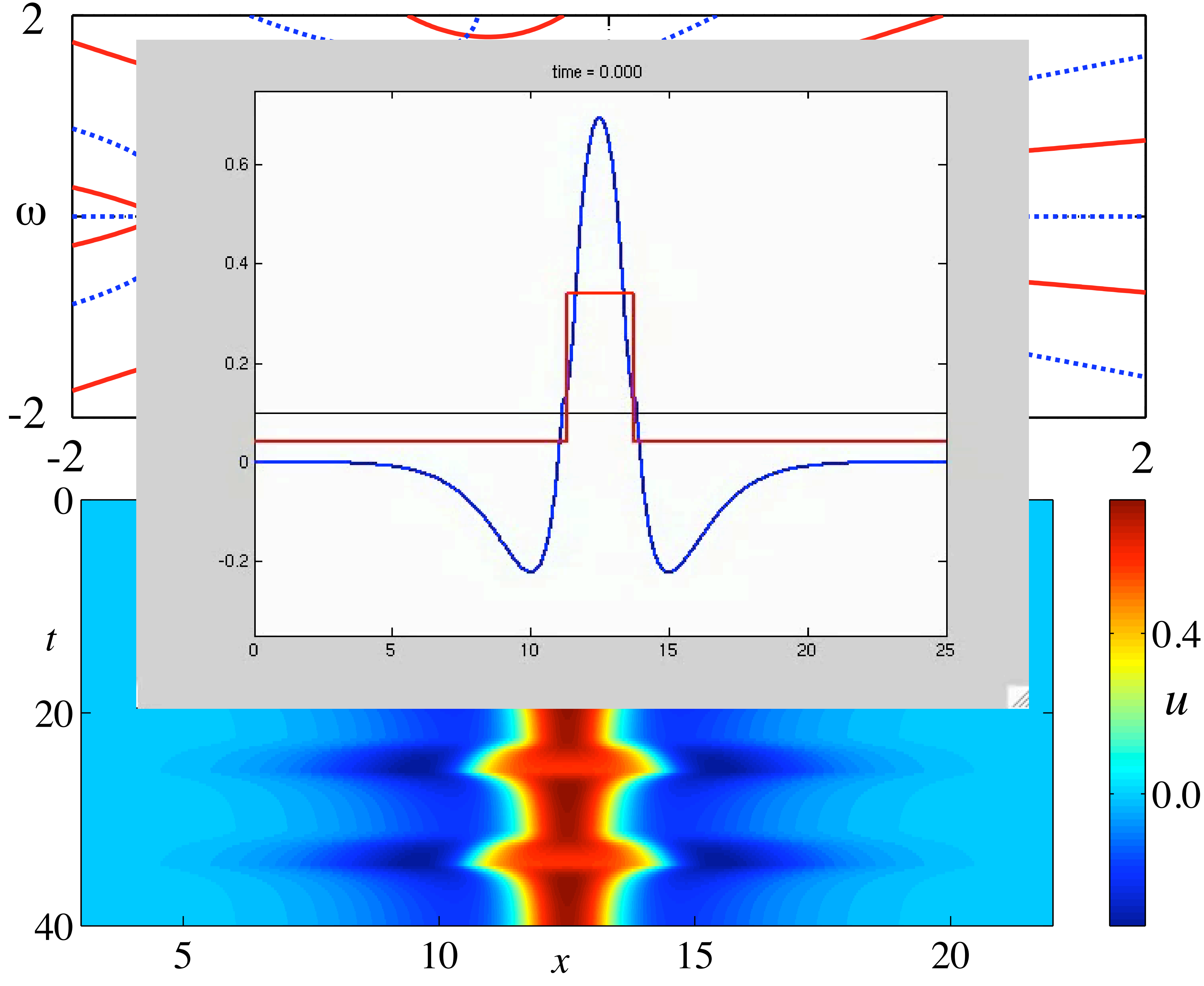
Bump Stability I: $\eta(t) = \alpha^2 t e^{-\alpha t}$

Low κ instability on Re axis (increasing α)

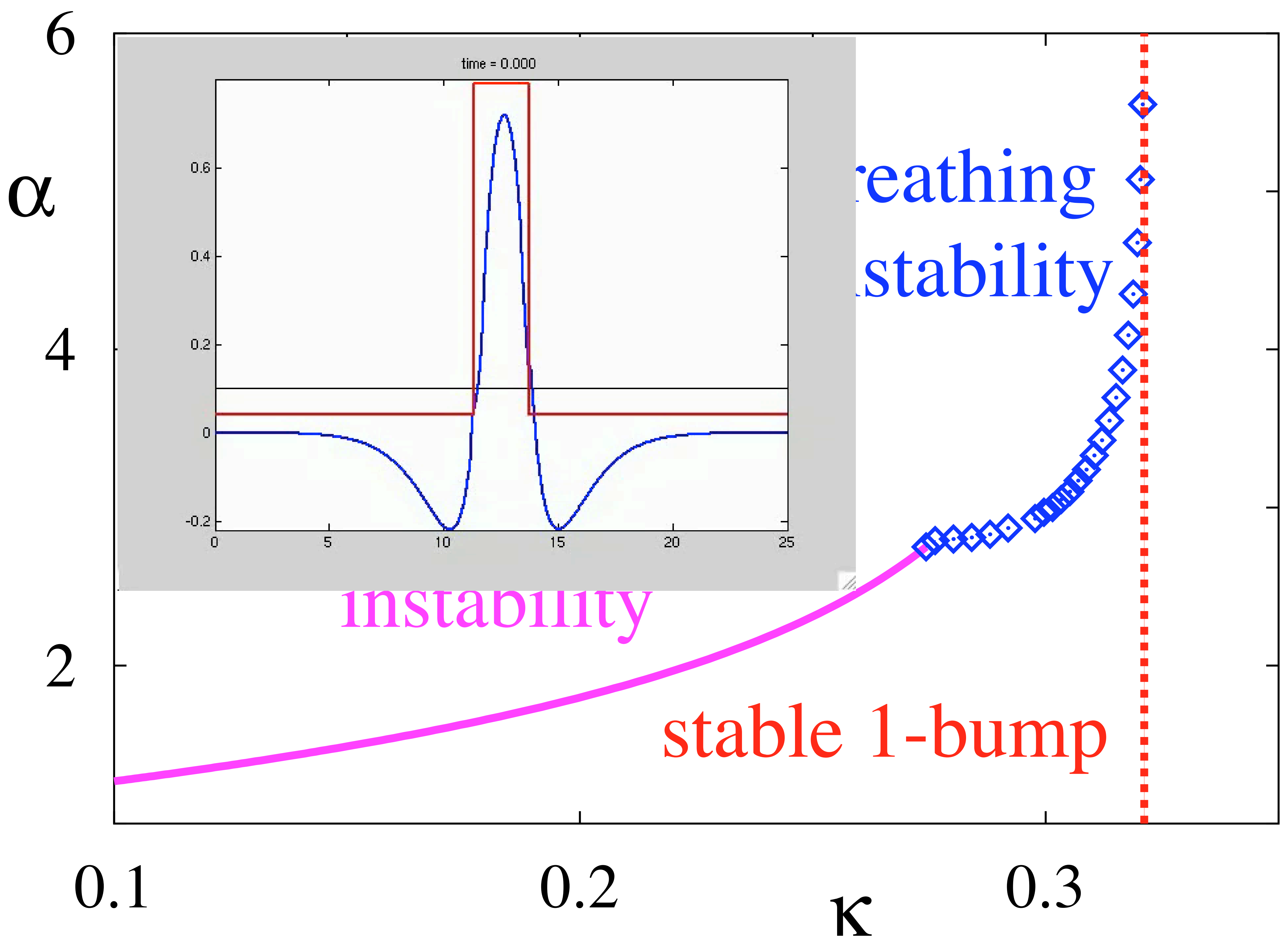


Bump Stability II

High κ instability on Im axis (increasing α) gives a breather

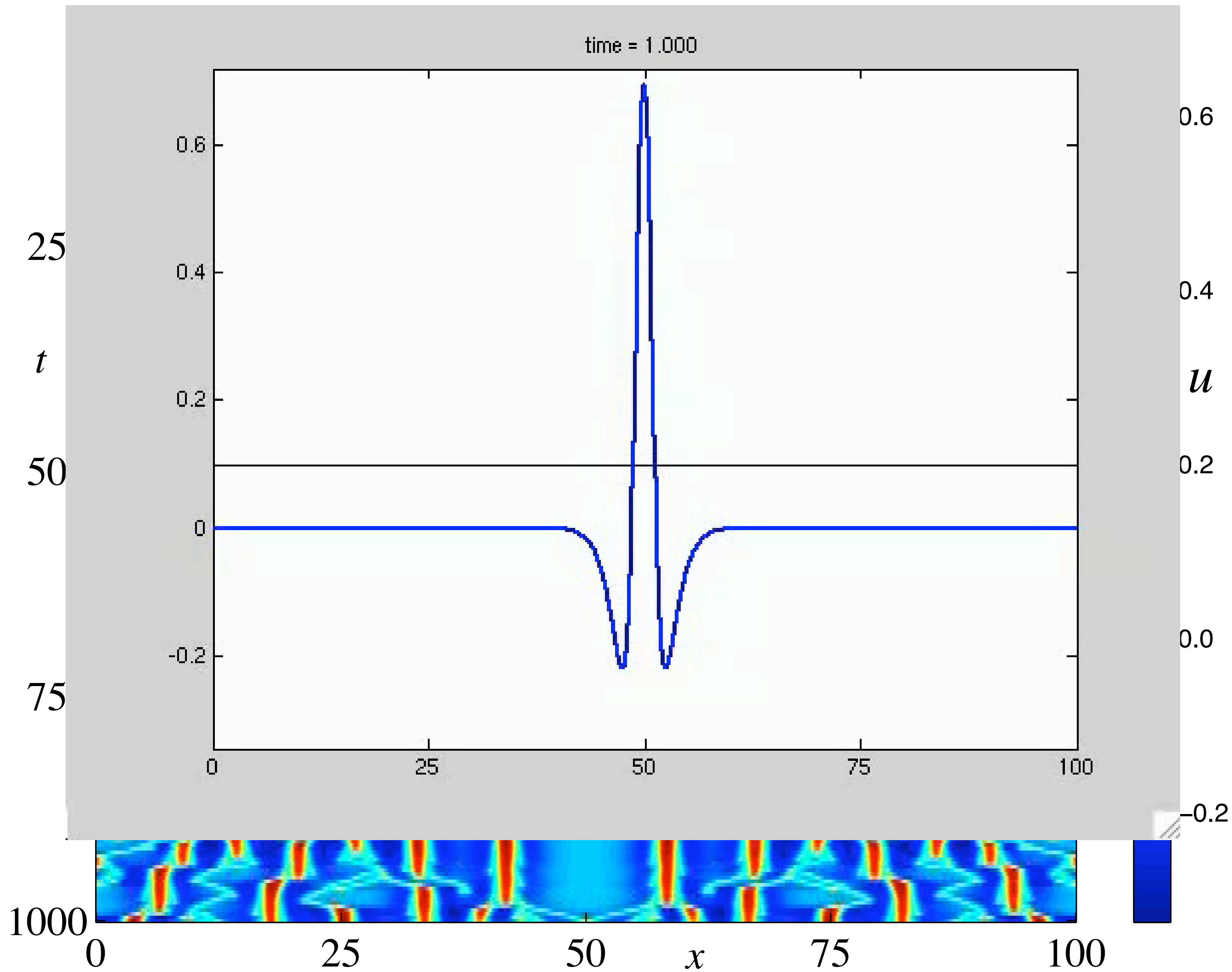


Summary of Bump instabilities



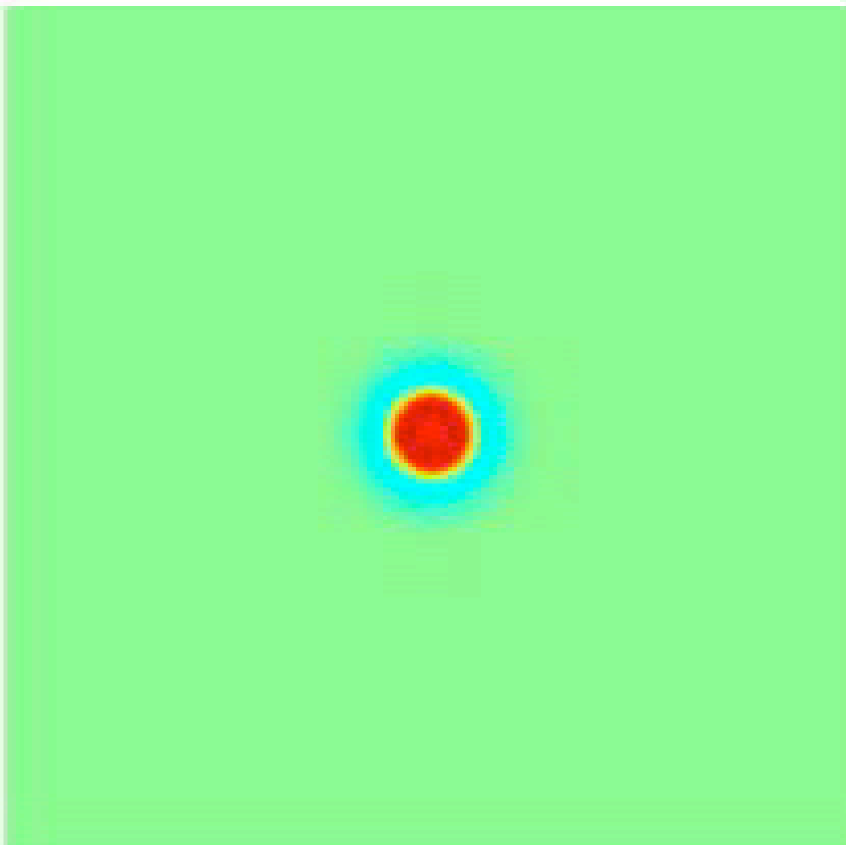
Exotic Dynamics

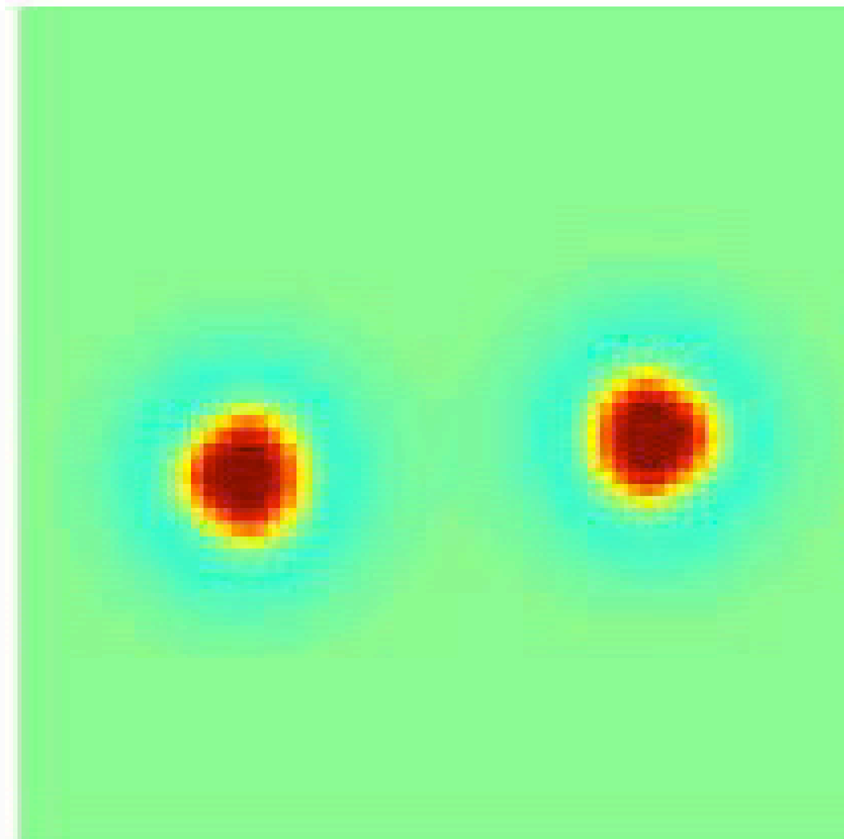
... including asymmetric breathers, multiple bumps, multiple pulses, periodic traveling waves, and bump-splitting instabilities that appear to lead to spatio-temporal chaos.



S Coombes and M R Owen: Bumps, breathers and waves in a neural network with spike frequency adaptation. PRL, 94, 148102, (2005).

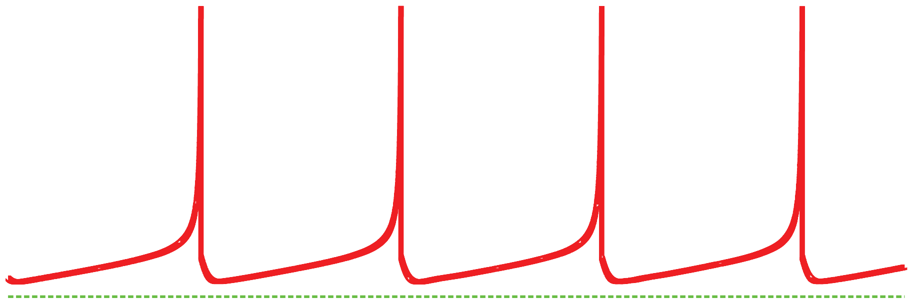
Splitting and scattering



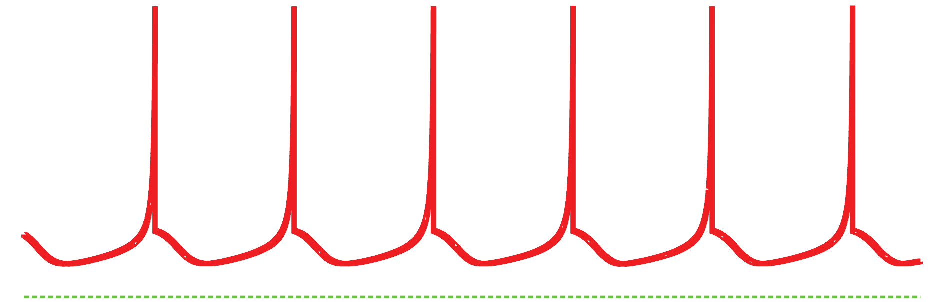


Auto/dispersive solitons as seen in coupled cubic complex Ginzburg-Landau systems and three component reaction-diffusion systems.

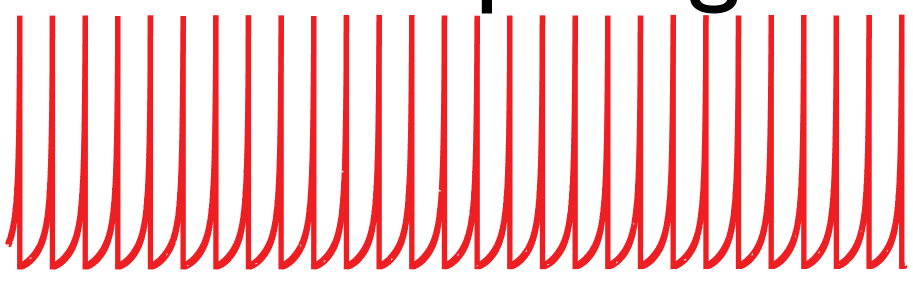
Regular spiking



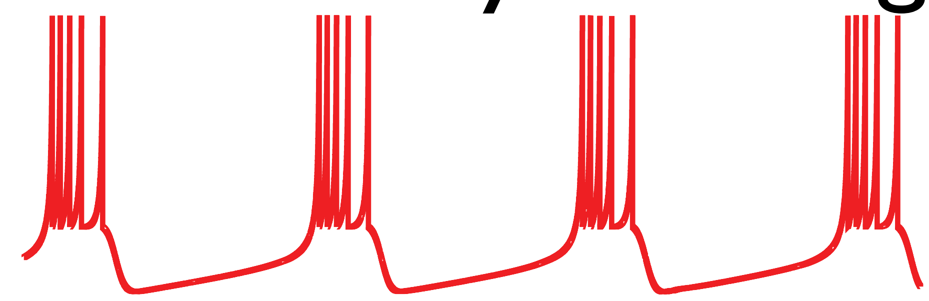
Chattering

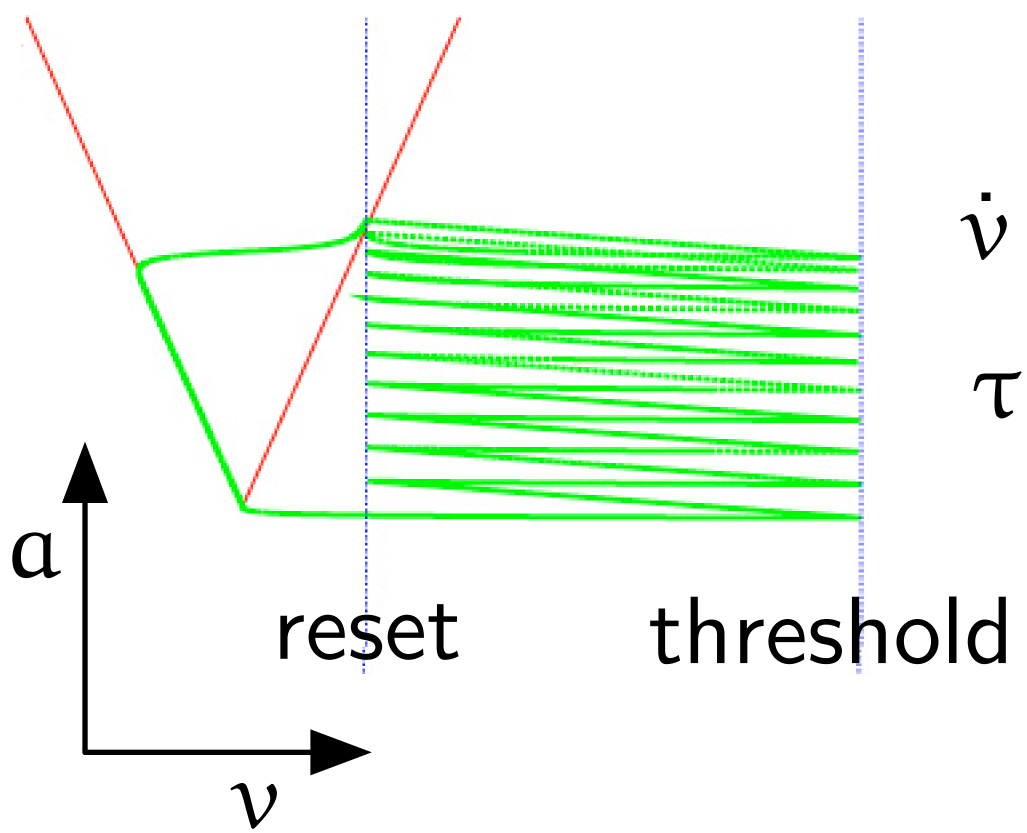


Fast spiking



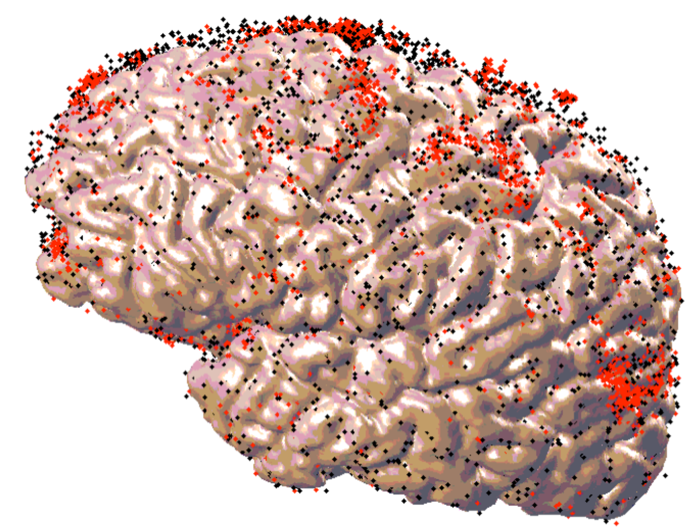
Intrinsically bursting





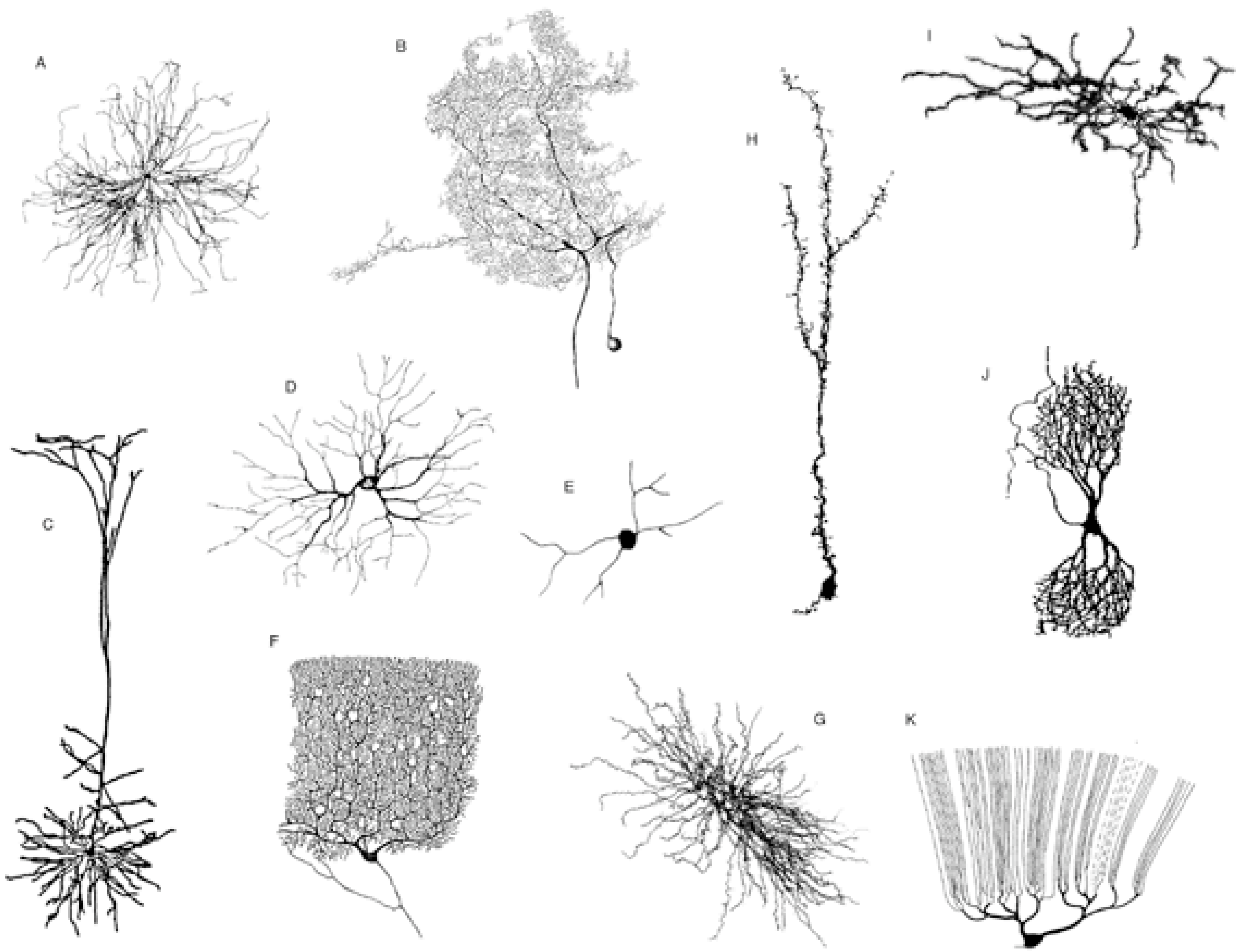
$$\dot{v} = |v| +$$

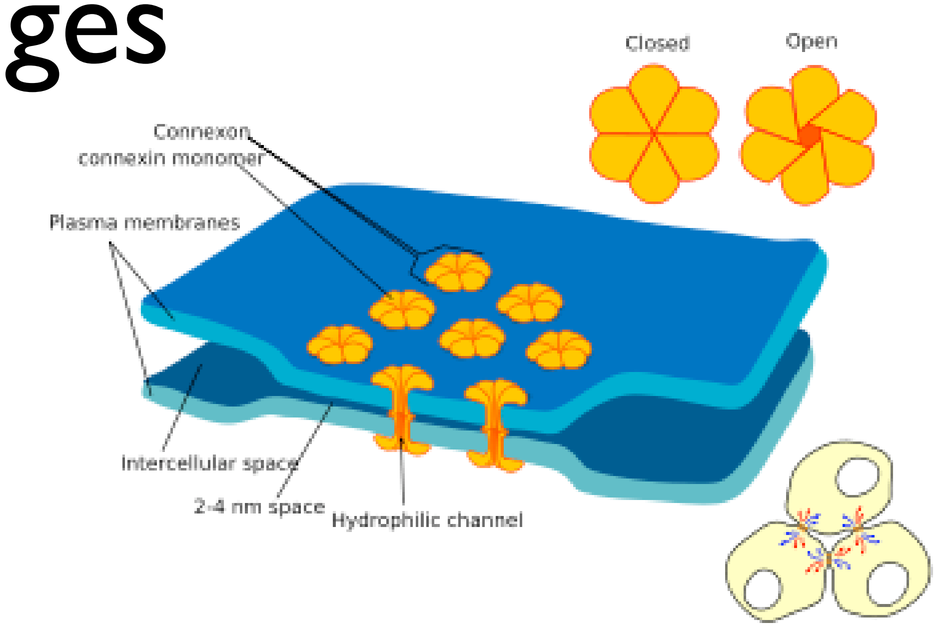
$$\tau \dot{a} = -a$$

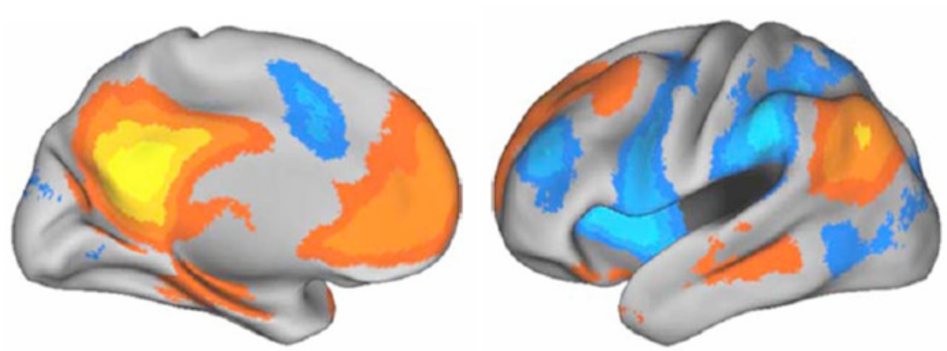


S Coombes and M Zachariou 2009, in Coherent Behavior in Neuronal Networks (Ed. Rubin, Josic, Matias, Romo), Springer.

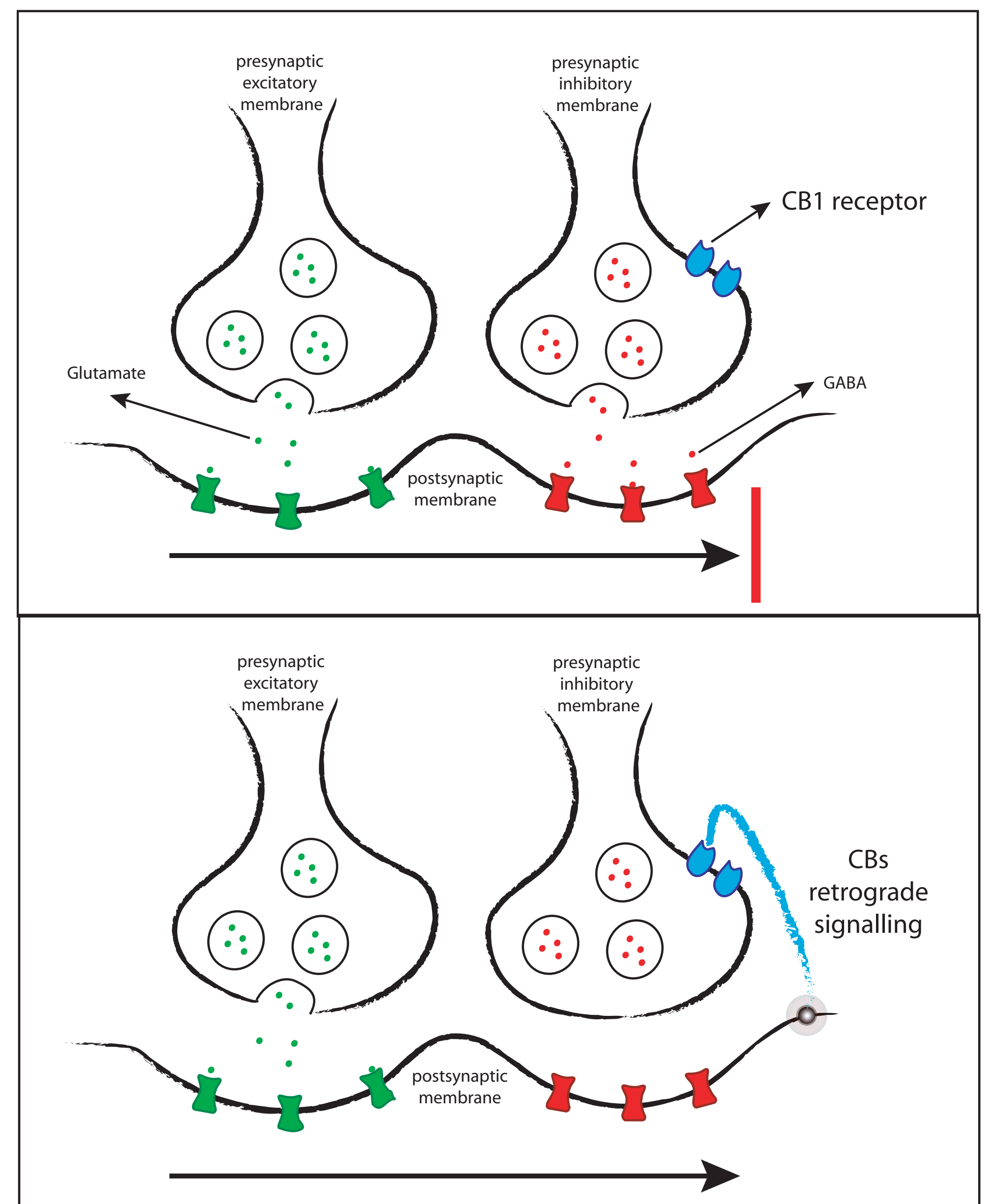
Further Challenges







Default mode network and ultra slow coherent oscillations



In collaboration with

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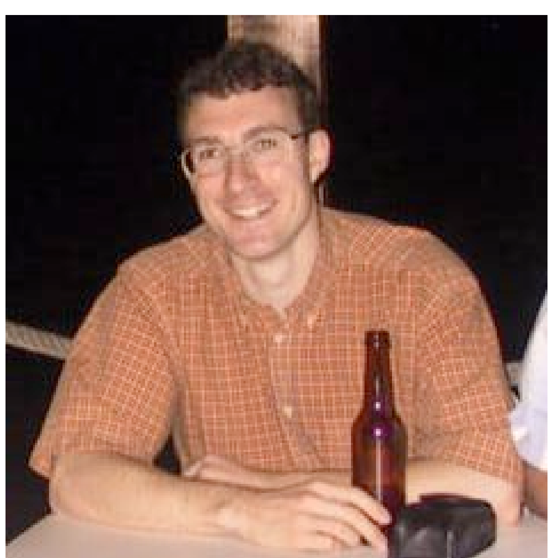
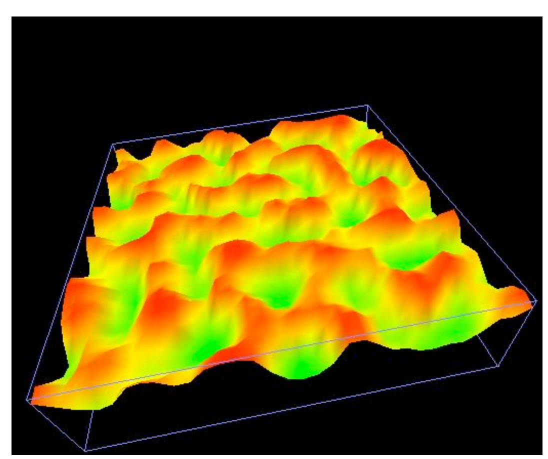
David Liley (Melbourne)

EPSRC Engineering and Physical Sciences Research Council

Markus Owen (Notts)



Ingo Bojak (Nijmegen)



Carlo Laing (Massey, NZ)